

SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

14/11/2024

1. An arithmetic progression (AP) is given as $600 + 650 + 700 + 750 + \dots$. Determine:
(a) the 30th term of the AP; (2 marks)

$$T_n = a + (n-1)d$$

$$a = 600, n = 30$$

$$d = 700 - 650 = 50 \checkmark$$

$$T_{30} = 600 + 50(30-1)$$

$$= 600 + (29 \times 50)$$

$$= 2050 \checkmark$$

- (b) the sum of the first 30 terms of the AP. (2 marks)

$$S_n = \frac{n}{2}[a + l]; l = \text{last term } \{30^{\text{th}} \text{ term}\}$$

$$S_{30} = \frac{30}{2}[600 + 2050] \checkmark$$

$$= 15 \times 2650$$

$$= 39750 \checkmark$$

Alternatively,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{30} = \frac{30}{2}[(600 \times 2) + 50(30-1)] \checkmark$$

$$= 15(1200 + 1450)$$

$$= 39750 \checkmark$$

2. The quadratic equation $5x^2 + kx + 20 = 0$ has only one root. Determine the possible values of k . (2 marks)

For a repeated root the discriminant, $b^2 - 4ac = 0$ hence $b^2 = 4ac$.

$$\therefore k^2 = 4 \times 5 \times 20 \checkmark$$

$$k^2 = 400$$

$$k = \pm\sqrt{400} = \pm 20 \text{ (20 or -20)} \checkmark$$



3. Without using mathematical tables or a calculator, evaluate $\frac{\log 125 + \log 64}{\log \sqrt[6]{5} + \log \sqrt[3]{2}}$. (3 marks)

$$125 = 5^3, 64 = 2^6, \sqrt[6]{5} = 5^{\frac{1}{6}}, \sqrt[3]{2} = 2^{\frac{1}{3}}$$

$$\frac{\log(5^3 \times 2^6)}{\log(5^{\frac{1}{6}} \times 2^{\frac{1}{3}})} \checkmark = \frac{\log(5^1 \times 2^2)^3}{\log(5^1 \times 2^2)^{\frac{1}{6}}} = \frac{3 \log(5^1 \times 2^2)^1}{\frac{1}{6} \log(5^1 \times 2^2)^1} \checkmark = \frac{3}{\frac{1}{6}} = 3 \times \frac{6}{1} = 18 \checkmark$$

Alternatively

$$\frac{\log(5^3 \times 2^6)}{\log(5^{\frac{1}{6}} \times 2^{\frac{1}{3}})} \checkmark = \frac{\log 5^3 + \log 2^6}{\log 5^{\frac{1}{6}} + \log 2^{\frac{1}{3}}} = \frac{3 \log 5 + 6 \log 2}{\frac{1}{6} \log 5 + \frac{1}{3} \log 2} \checkmark = \frac{3(\log 5 + 2 \log 2)^1}{\frac{1}{6}(\log 5 + 2 \log 2)^1} = \frac{3}{\frac{1}{6}} = 3 \times \frac{6}{1} = 18 \checkmark$$

4. Make x the subject of the formula $y = \frac{a}{b^x}$. (3 marks)

$$yb^x = a \Rightarrow b^x = \frac{a}{y} \checkmark$$

Taking logs on both sides:

$$x \log b = \log \left(\frac{a}{y} \right) \checkmark \text{ hence } x = \frac{\log \left(\frac{a}{y} \right)}{\log b} \checkmark$$

Alternatively

$$yb^x = a \Rightarrow b^x = \frac{a}{y} \checkmark$$

Taking logs on both sides:

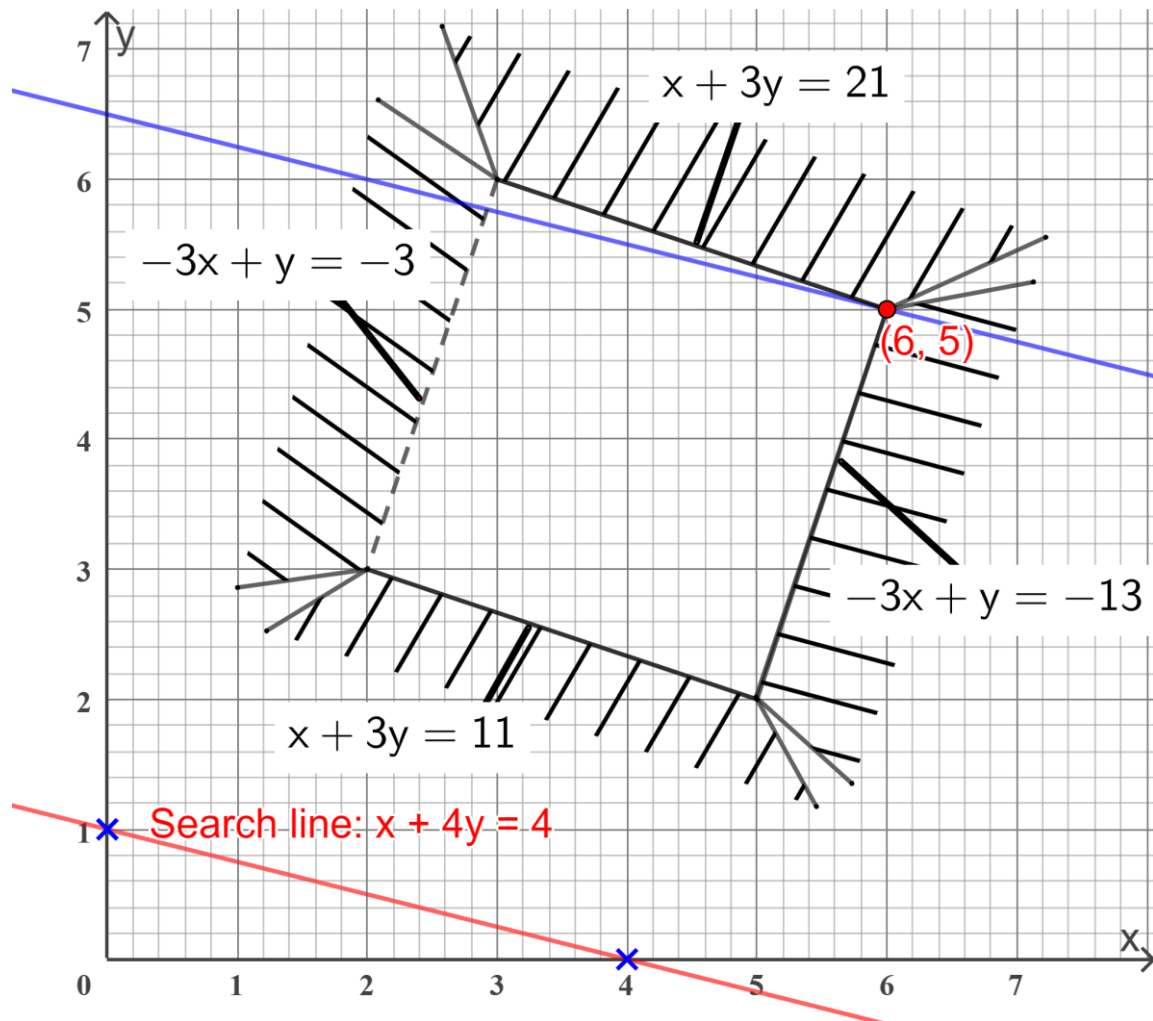
$$x \log b = \log \left(\frac{a}{y} \right)$$

$$x \log b = \log a - \log y \checkmark$$

$$x = \frac{\log a - \log y}{\log b} \checkmark \text{ or } x = \frac{\log a}{\log b} - 1 \checkmark$$

5. The unshaded region on the Cartesian plane satisfies the inequalities

$$x + 3y \leq 21, -3x + y < -3, -3x + y \geq -13 \text{ and } x + 3y \geq 11.$$



Find the maximum value of $(x + 4y)$ for the integral coordinates $P(x, y)$ lying in the unshaded region. (3 marks)

Objective function: $x + 4y = M$ **Let** $x + 4y = 4$. **Mark and join points (4, 0) and (0, 1) to obtain the search line.** ✓ **Using a ruler and set square only, produce a parallel line to the search line at the last plausible point (6, 5) ✓ before leaving the feasible region. $M = 6 + (4 \times 5) = 26$ ✓**

6. An aircraft flew due west from point A ($39.64^\circ\text{N}, 50^\circ\text{E}$) to B ($39.64^\circ\text{N}, 20^\circ\text{W}$). Calculate the distance covered by the aircraft correct to the nearest km. (Take $\pi = \frac{22}{7}$ and $R = 6370 \text{ km}$) (3 marks)

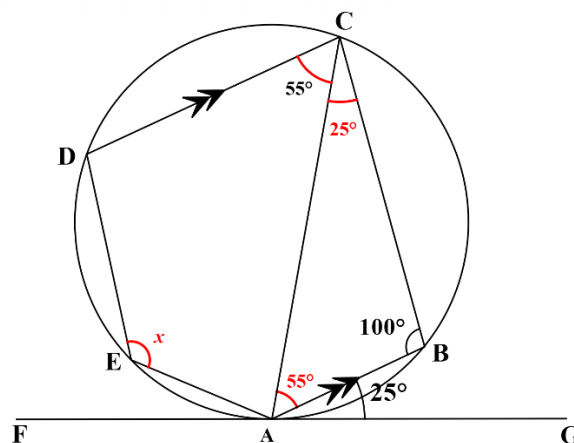
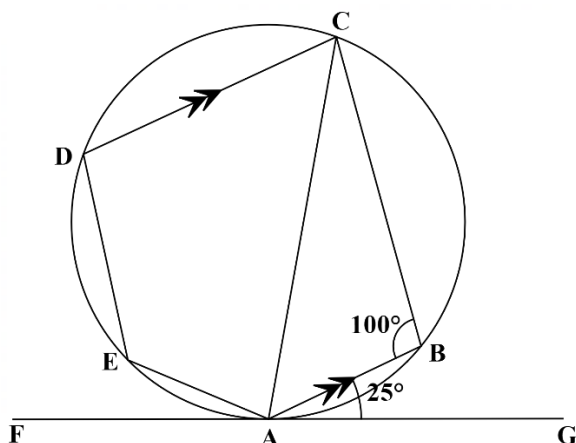
Longitude difference, $\theta = 50^\circ + 20^\circ = 70^\circ$ ✓ (A and B are on either side of the prime meridian)

β = latitude angle, R = Radius of the earth = 6370 km

$$\begin{aligned} \text{Distance along a parallel of latitude} &= \frac{\theta}{360^\circ} \times 2\pi R \cos \beta \\ &= \frac{70^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 6370 \times \cos 39.64^\circ \text{ ✓} \\ &= \frac{70070}{9} \times 0.7701 \\ &= 5995.6563 \text{ km} \\ &\approx 5996 \text{ km} \text{ ✓} \end{aligned}$$



7. In the following figure; A, B, C, D and E are points on the circumference of the circle. Line AB is parallel to line DC and line FAG is tangent to the circle at A. Angle GAB = 25° and $\angle ABC = 100^\circ$.



Determine the size of:

- (a) $\angle BAC$; (1 mark)

$$\angle BCA = \angle GAB = 25^\circ \text{ (by alternate segment//tangent-chord theorem)}$$

$$180^\circ - (100^\circ + 25^\circ) = 55^\circ \checkmark$$

- (b) $\angle AED$. (2 marks)

Since $AB \parallel CD$, AC is a transversal. Thus, $\angle BAC = \angle ACD = 55^\circ$ (alternate angles) $\checkmark$

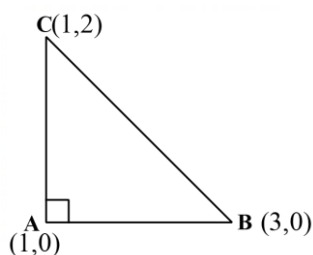
Opposite angles of cyclic quadrilateral ACDE are supplementary (add up to 180° .)

$$\therefore x = 180^\circ - 55^\circ = 125^\circ \checkmark$$

8. The triangle ABC with vertices A(1,0), B(3,0) and C(1,2) is transformed by the matrix

$$\mathbf{T} = \begin{pmatrix} 3k & 1.6 \\ 3k & -0.9 \end{pmatrix} \text{ onto triangle } A'B'C'. \text{ Given that the area of triangle } A'B'C' \text{ is 6 square units,}$$

determine the value of k. (3 marks)



$$\text{Area of } \triangle ABC = \frac{1}{2} \times 2 \times 2 = 2 \text{ sq. units}$$

$$\begin{aligned} \text{ASF} &= \frac{\text{Area of } \triangle A'B'C'}{\text{Area of } \triangle ABC} \\ &= \frac{6}{2} \\ &= 3 \checkmark \end{aligned}$$

$$\begin{aligned} \text{Det } \mathbf{T} &= \begin{vmatrix} 3k & 1.6 \\ 3k & -0.9 \end{vmatrix} = 3k(-0.9) - 3k(1.6) \\ &= -2.7k - 4.8k \\ &= -7.5k \checkmark \end{aligned}$$

$$\boxed{\text{Det } \mathbf{T} = \text{ASF}} \text{ hence } -7.5k = 3$$

$$k = \frac{3}{-7.5} = -\frac{1}{4} \text{ or } -0.25 \checkmark$$

9. Solve the equation $6\cos^2 x + \sin x = 4$ for $0^\circ \leq x \leq 180^\circ$, giving the answer correct to 2 decimal places. (3 marks)

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \boxed{\cos^2 x = 1 - \sin^2 x}$$

$$6(1 - \sin^2 x) + \sin x = 4$$

$$6 - 6\sin^2 x + \sin x = 4$$

$$6\sin^2 x - \sin x - 2 = 0 \checkmark$$

$$6\sin^2 x + 3\sin x - 4\sin x - 2 = 0$$

$$3\sin x(2\sin x + 1) - 2(2\sin x + 1) = 0$$

$$(3\sin x - 2)(2\sin x + 1) = 0 \checkmark$$

$$3\sin x - 2 = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$

$$3\sin x = 2 \quad \quad \quad 2\sin x = -1$$

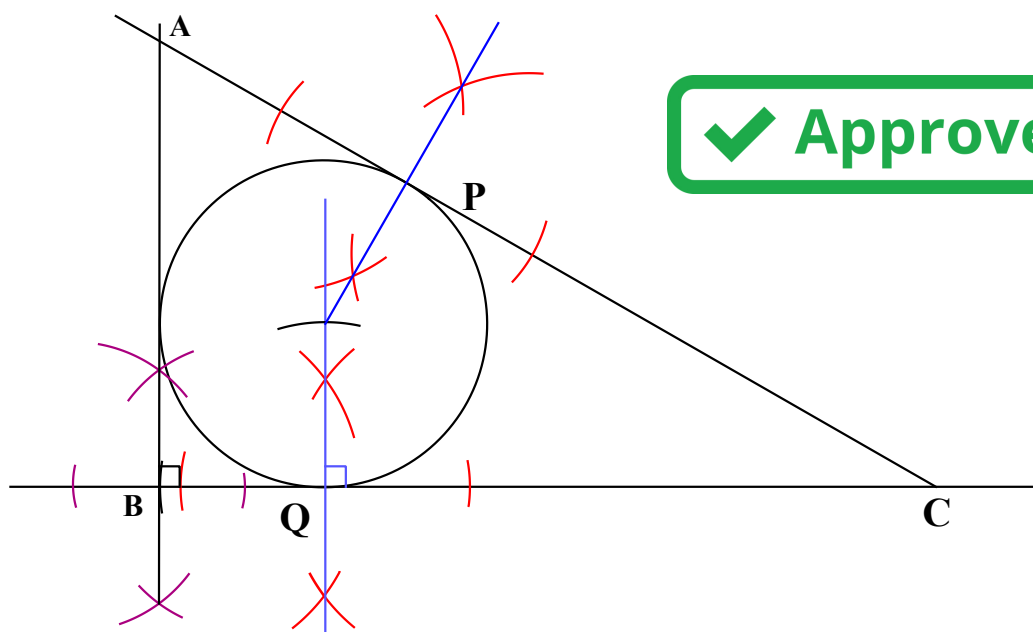
$$\sin x = 0.6667 \quad \quad \quad \sin x = -0.5$$

Since sine is positive in the range $0^\circ \leq x \leq 180^\circ$;

$$x = \sin^{-1} 0.6667$$

$$= 41.81^\circ, 138.19^\circ \checkmark$$

10. The figure below shows a circle. Lines CP and CQ are tangents to the circle at points P and Q respectively.



The circle is to be inscribed in a triangle ABC. Point B lies on CQ produced and $\angle CBA = 90^\circ$. Use a ruler and a pair of compasses only to:

- locate point O, the centre of the circle; (2 marks)
- complete triangle ABC. (2 marks)

11. The deviations of the masses of 10 students from an assumed mean are:

$-10, -5, -2, 1, 4, 5, 7, 8, 9, 13$

The mass of the heaviest student was 58 kg. Calculate the mean mass of the students. (3 marks)

Let the assumed mean be A. $\therefore 58 - A = 13 \Rightarrow A = 58 - 13 = 45$ ✓

t	-10	-5	-2	1	4	5	7	8	9	13	
f	1	1	1	1	1	1	1	1	1	1	$\sum f = 10$
ft	-10	-5	-2	1	4	5	7	8	9	13	$\sum ft = 30$ ✓

$$\bar{x} = A + \frac{\sum ft}{\sum f} = 45 + \frac{30}{10} = 48$$
✓

12. The following table shows part of a monthly income tax rates for a certain year.

Monthly taxable income (Ksh.)	Tax rate (%)
0 to 11 180	10
11 181 to 21 714	15
21 715 to 32 248	20

In a certain month an employee paid a net tax of Ksh. 2 200 after getting a tax relief of Ksh. 1 280.

Calculate the employee's taxable income that month. (3 marks)

$$\text{Gross tax : } \left(\frac{10}{100} \times 11180 \right) + \left(\frac{15}{100} \times 10534 \right) + \left(\frac{20}{100} \times y \right) = 2200 + 1280$$
✓

$$1118 + 1580.10 + 0.2y = 3480$$

$$0.2y = 781.9$$

$$y = \text{Ksh. } 3909.50$$
✓

$$\text{Taxable income} = \text{Ksh. } (21714 + 3909.50) = \underline{\underline{\text{Ksh. } 25\,623.50}}$$
✓

13. The equation of a circle is given by $x^2 + y^2 - 3x + 4y = 0$. Determine:

(a) the coordinates of the centre of the circle;

(2 marks)

Compare $x^2 + y^2 - 3x + 4y = 0$ with $\downarrow$ $x^2 + y^2 - 2ax - 2by + [a^2 + b^2 - r^2] = 0$ By comparing : $-2a = -3 \Rightarrow a = \frac{-3}{-2} = 1.5$ $-2b = 4 \Rightarrow b = -\frac{4}{2} = -2$ } ✓ Centre (1.5, -2) ✓	Alternatively Rearranging: $x^2 - 3x + y^2 + 4y = 0$ $x^2 - 3x + \left(\frac{-3}{2}\right)^2 + y^2 + 4y + \left(\frac{4}{2}\right)^2 = \left(\frac{-3}{2}\right)^2 + \left(\frac{4}{2}\right)^2$ ✓ $(x - 1.5)^2 + (y + 2)^2 = 6.25$ $(x - a)^2 + (y - b)^2 = r^2$ $-a = -1.5 \Rightarrow a = 1.5, \quad -b = 2 \Rightarrow b = -2$ Centre (1.5, -2) ✓
---	--

(b) the area of the circle in terms of π .

(1 mark)

$$r^2 = 6.25 \text{ hence area of circle} = \pi r^2 = \underline{\underline{6.25\pi \text{ sq.units}}} \checkmark$$

or using $a^2 + b^2 - r^2 = c, \quad r^2 = a^2 + b^2 - c$

$$r^2 = 1.5^2 + (-2)^2 - 0$$

$$= 6.25$$

$$\text{Area of circle} = \pi r^2 = \underline{\underline{6.25\pi \text{ sq.units}}} \checkmark$$

14. The position vectors of points A, B and C are such that $\mathbf{OA} = 3\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$, $\mathbf{OB} = \mathbf{j} + 8\mathbf{k}$ and $\mathbf{OC} = -2\mathbf{i} + 5\mathbf{j} + 16\mathbf{k}$. Show that the points A, B and C are collinear.

(3 marks)

Let $\mathbf{AC}$ and $\mathbf{AB}$ be 2 vectors such that $\mathbf{AC} = k\mathbf{AB}$. $\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = (-2\mathbf{i} + 5\mathbf{j} + 16\mathbf{k}) - (3\mathbf{i} - 5\mathbf{j} - 4\mathbf{k})$ $= -5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k}$ $\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (\mathbf{j} + 8\mathbf{k}) - (3\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}) \checkmark$ $= -3\mathbf{i} + 6\mathbf{j} + 12\mathbf{k}$ $-5\mathbf{i} + 10\mathbf{j} + 20\mathbf{k} = k(-3\mathbf{i} + 6\mathbf{j} + 12\mathbf{k})$ $k = \frac{-5}{-3} = \frac{10}{6} = \frac{20}{12} = \frac{5}{3} \checkmark$ $\mathbf{AC} = \frac{5}{3}\mathbf{AB} \text{ hence } \mathbf{AC} \parallel \mathbf{AB} \text{ and A is a common point therefore A, B and C are collinear.} \checkmark$	Alternatively $\mathbf{AC} = \begin{pmatrix} -2 \\ 5 \\ 16 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 20 \end{pmatrix}$ $\mathbf{AB} = \begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \\ -4 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ 12 \end{pmatrix}$ $\begin{pmatrix} -5 \\ 10 \\ 20 \end{pmatrix} = k \begin{pmatrix} -3 \\ 6 \\ 12 \end{pmatrix} \Rightarrow k = \frac{-5}{-3} = \frac{10}{6} = \frac{20}{12} = \frac{5}{3} \checkmark$ $\mathbf{AC} = \frac{5}{3}\mathbf{AB} \text{ hence } \mathbf{AC} \parallel \mathbf{AB} \text{ and A is a common point therefore A, B and C are collinear.} \checkmark$
--	---

15. A particle starts from point O and moves in a straight line so that its velocity $v \text{ ms}^{-1}$ after time t seconds is given by $v = 9t^2 - 18t + 10$. Calculate the distance covered by the particle between the time $t = 1$ second and $t = 2$ seconds.

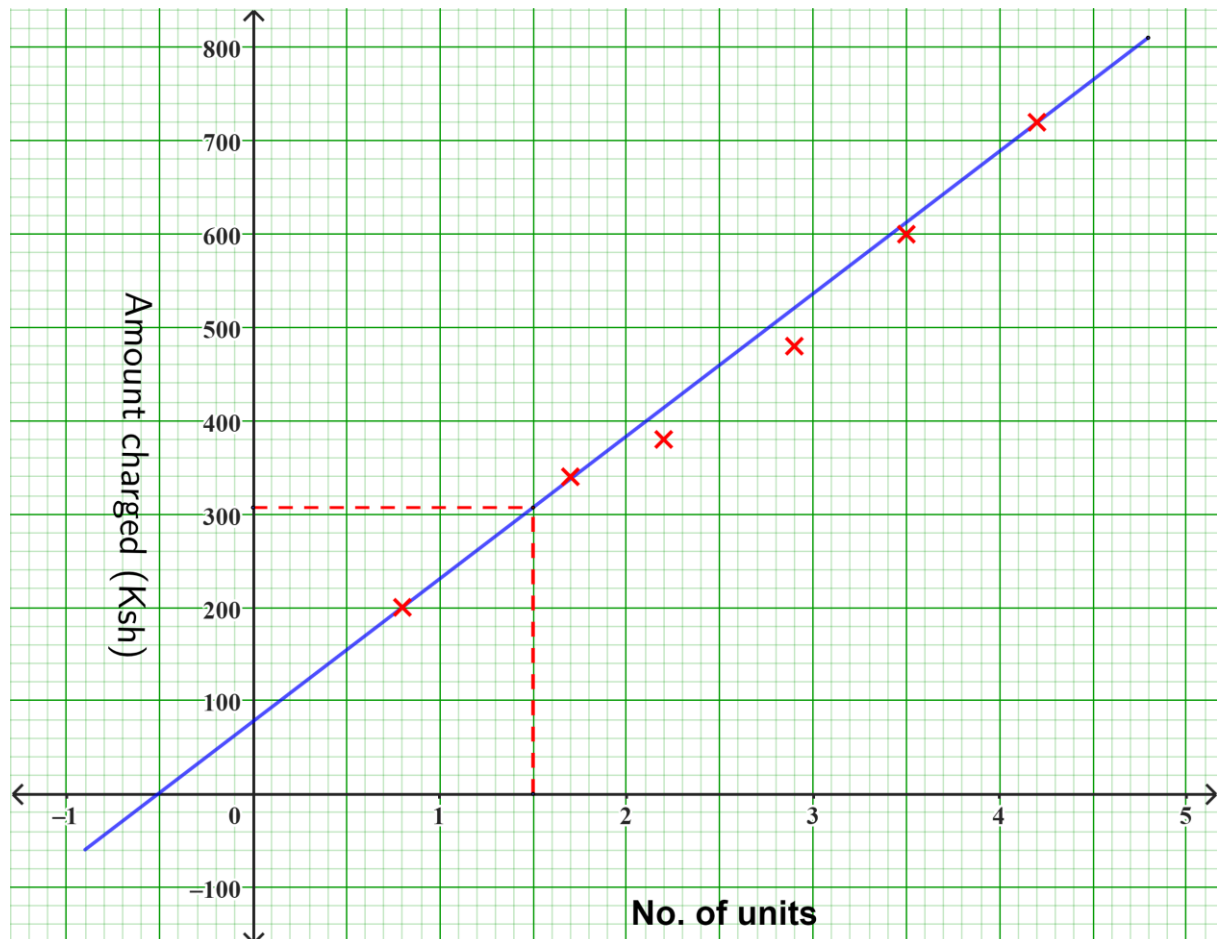
(3 marks)

$$\begin{aligned}
 S &= \int_1^2 (9t^2 - 18t + 10) dt \\
 &= \left[\frac{9t^{2+1}}{3} - \frac{18t^{1+1}}{2} + \frac{10t^{0+1}}{0+1} \right]_1^2 \checkmark \\
 &= [3t^3 - 9t^2 + 10t]_1^2 \\
 &= [3(2^3) - 9(2^2) + 10(2)] - [3(1^3) - 9(1^2) + 10(1)] \checkmark \\
 &= 8 - 4 \\
 &= \underline{\underline{4 \text{ metres}}} \checkmark
 \end{aligned}$$

16. The following table shows the number of units (U) of water consumed by 6 households in a month. The corresponding amount (A) charged is also given.

No. of units (U)	0.8	1.7	2.2	2.9	3.5	4.2
Amount (A) charged in Ksh.	200	340	380	480	600	720

- (a) Using the scale 2 cm to represent 1 unit on the x -axis and 1 cm to represent Ksh. 100 on the y -axis, draw the line of best fit for the data on the grid provided. (3 marks)



- (b) Estimate the cost of 1.5 units of water. (1 mark)

≈ Ksh. 310✓



SECTION II (50 marks)

Answer only **five** questions in this section in the spaces provided.

17. A poultry dealer has two types of chicken feeds: type A and type B. He sells 1 kg of type A at Ksh. 45 and 1 kg type B at Ksh. 30. He makes a profit of 20% per kg of type A feed sold and 25% per kg of type B feed sold. He also sells mixtures of type A and type B feeds.

- (a) Determine the amount of profit made by the dealer for selling 1 kg of:

- (i) type A feed. (1 mark)

$$\text{Profit} = \frac{20}{100} \times 45 = \underline{\text{Ksh. 7.50}} \quad \text{OR} \quad \text{Profit} = 45 - \left(\frac{100 \times 45}{120} \right) = \underline{\text{Ksh. 7.50}}$$

- (ii) type B feed. (1 mark)

$$\text{Profit} = \frac{25}{100} \times 30 = \underline{\text{Ksh. 6}} \quad \text{OR} \quad \text{Profit} = 30 - \left(\frac{100 \times 30}{125} \right) = \underline{\text{Ksh. 6}}$$

- (b) Type A and type B feeds were mixed in the ratio 3:7. Calculate:

- (i) the selling price of 1 kg of the mixture; (2 marks)

$$\begin{aligned} \text{Selling price (per kg of mixture)} &= \frac{(3 \times 45) + (7 \times 30)}{3 + 7} = \frac{345}{10} \\ &= \underline{\text{Ksh. 34.50}} \end{aligned}$$

- (ii) the profit made by the dealer in selling 50 kg of the mixture. (2 marks)

$$\text{Profit in selling 1 kg of mixture} = \frac{(3 \times 7.50) + (7 \times 6)}{10} = \text{Ksh. 6.45}$$

$$\text{Profit in selling 50 kg of mixture} = 50 \times 6.45 = \underline{\text{Ksh. 322.50}}$$

- (c) The dealer made a profit of Ksh. 1 387.50 for the sale of 200 kg of a different mixture of type A and type B feeds. Determine the ratio of type A feed to that of type B feed in the mixture. (4 marks)

Let the ratio of A:B in the mixture be $x : y$.

$$200 \left(\frac{7.50x + 6y}{x + y} \right) = 1387.50$$

Dividing both sides by 200...

$$\frac{7.50x + 6y}{x + y} = \frac{111}{16}$$

$$16(7.50x + 6y) = 111(x + y)$$

$$120x + 96y = 111x + 111y$$

$$9x = 15y$$

Dividing both sides by 9y...

$$\frac{x}{y} = \frac{5}{3}$$

$$\therefore \underline{\underline{\text{A:B} = 5:3}}$$

18.

- (a) A quantity P is partly constant and partly varies as the square root of a quantity Q. Given that P = 20 when Q = 4 and that P = 60 when Q = 100, find Q when P = 22. (4 marks)

$P = C + k\sqrt{Q}$ $20 = C + k\sqrt{4} \Rightarrow 20 = C + 2k \dots(i)$ $60 = C + k\sqrt{100} \Rightarrow 60 = C + 10k \dots(ii)$ $(ii) - (i): 8k = 40$ $k = 5$ $\text{Using (i), } C = 20 - (2 \times 5) = 10$ $\therefore \boxed{P = 10 + 5\sqrt{Q}}$	$\text{When } P = 22; \quad 5\sqrt{Q} = 22 - 10$ $5\sqrt{Q} = 12$ $\sqrt{Q} = 2.4$ $Q = 2.4^2$ $= 5.76$
---	---

- (b) Three quantities, T, U and V are such that T varies directly as the square of $(10 - U)$ and inversely as the cube root of V. When T = 12, U = 4 and V = 8.

- (i) Determine the equation connecting T, U and V. (3 marks)

$$T \propto \frac{(10 - U)^2}{\sqrt[3]{V}}$$

$$\therefore T = \frac{k(10 - U)^2}{\sqrt[3]{V}}$$

Given T = 12 when U = 4 and V = 8;

$$12 = \frac{k(10 - 4)^2}{\sqrt[3]{8}}$$

$$12 = \frac{k \times 6^2}{2}$$

$$36k = 24$$

$$k = \frac{24}{36} = \frac{2}{3}$$

$$\therefore \text{the equation: } \boxed{T = \frac{\frac{2}{3}(10 - U)^2}{\sqrt[3]{V}}} \text{ or } \boxed{T = \frac{2(10 - U)^2}{3\sqrt[3]{V}}}$$

- (ii) Find U when $T = 5\frac{2}{5}$ and $V = 15\frac{5}{8}$. (3 marks)

$$\frac{27}{5} = \frac{2(10 - U)^2}{3 \times \sqrt[3]{15.625}}$$

$$27 \times 3 \times \sqrt[3]{15.625} = 5 \times 2(10 - U)^2$$

$$27 \times 3 \times 2.5 = 10(10 - U)^2$$

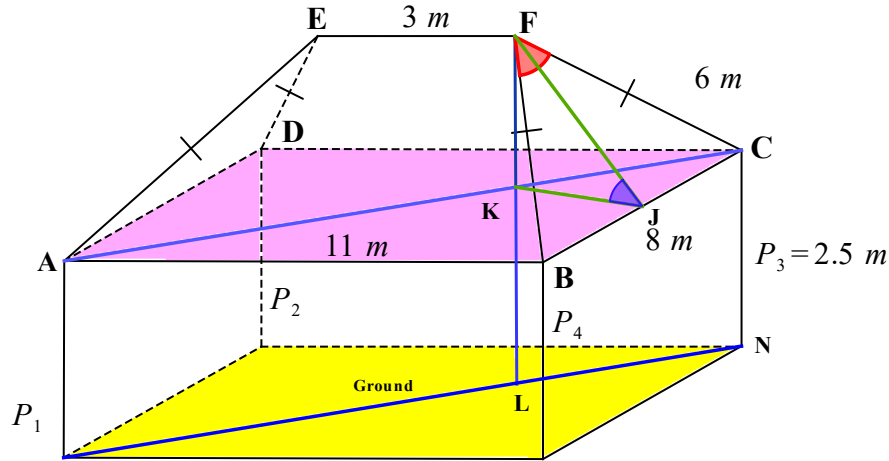
$$\frac{27 \times 3 \times 2.5}{10} = 10^1 (10 - U)^2 \times \frac{1}{10^1}$$

$$\sqrt{20.25} = \sqrt{(10 - U)^2}$$

$$\pm 4.5 = 10 - U$$

$$\therefore U = 10 - 4.5 = 5.5 \text{ Or } U = 10 + 4.5 = 14.5$$

19. The following figure shows a tent erected on a level ground. The roof ABCDEF of the tent is supported by four vertical posts P_1 , P_2 , P_3 and P_4 each of height 2.5 m. The ridge EF = 3 m is centrally placed. Further, AB = 11 m, BC = 8 m and FB = FC = ED = EA = 6 m.



Calculate:

- (a) the length of the projection of FC on the ground correct to 4 significant figures. (3 marks)

$$KJ = \frac{11-3}{2} = 4m \checkmark \quad | \quad BJ = JC = \frac{8}{2} = 4m$$

Plane ABCD = Rectangular part of the ground formed by the supporting posts then translated.

The projection of FC on the ground is LN.

$$\text{But } LN = KC$$

$$\begin{aligned} KC &= \sqrt{KJ^2 + JC^2} \\ &= \sqrt{4^2 + 4^2} \checkmark \\ &= \underline{\underline{5.657 \text{ m}}} \checkmark \end{aligned}$$

- (b) the height of the ridge EF above the ground. (3 marks)

Required length is FL.

$$FL = FK + KL$$

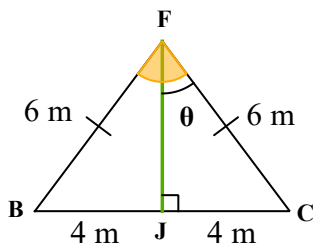
$$= FK + 2.5m$$

Using $\triangle FKC$;

$$\begin{aligned} FK &= \sqrt{FC^2 - KC^2} = \sqrt{6^2 - 32} \checkmark \\ &= 2m \checkmark \end{aligned}$$

$$\begin{aligned} FL &= 2m + 2.5m \\ &= \underline{\underline{4.5 \text{ m}}} \checkmark \end{aligned}$$

- (c) the angle between edge FB and edge FC. (2 marks)



Required angle is $\angle BFC$.

$$\sin \theta = \frac{4}{6} \Rightarrow \theta = \sin^{-1} 0.6667$$

$$= 41.81^\circ \checkmark$$

$$\angle BFC = 2 \times 41.81^\circ$$

$$= \underline{\underline{83.62^\circ}} \checkmark$$

Alternatively, using cosine rule;

$$8^2 = 6^2 + 6^2 - (2 \times 6 \times 6 \cos F)$$

$$\angle F = \cos^{-1} \left(\frac{6^2 + 6^2 - 8^2}{2 \times 6 \times 6} \right) \checkmark$$

$$= \cos^{-1} 0.1111$$

$$= \underline{\underline{83.62^\circ}} \checkmark$$

- (d) the angle between the plane FBC and the ground. (2 marks)

Angle between the plane FBC and the ground = angle between the plane FBC and plane ABCD

= angle between FJ and its projection JK on ABCD

$$\text{Using triangle FKJ; } \tan J = \frac{FK}{KJ} \Rightarrow J = \tan^{-1} \left(\frac{FK}{KJ} \right) = \tan^{-1} \left(\frac{2}{4} \right) \checkmark$$

$$= \tan^{-1} 0.5$$

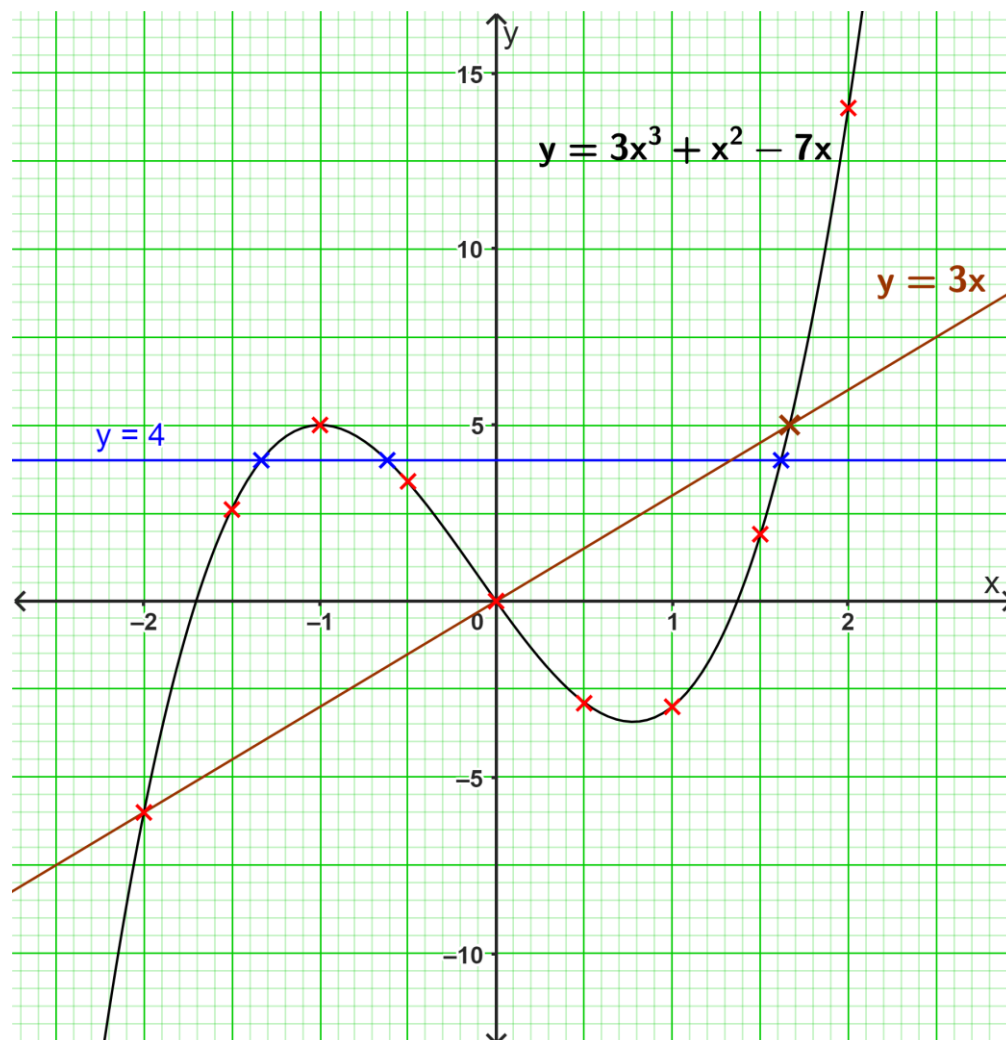
$$= \underline{\underline{26.57^\circ}} \checkmark$$

20. The table below shows values of x and some values of y for the curve $y = 3x^3 + x^2 - 7x$ in the range $-2 \leq x \leq 2$.

(a) Complete the table by filling the missing values of y correct to 1 decimal place. (2 marks)

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-6	2.6	5	3.4	0	-2.9	-3	1.9	14

(b) On the grid provided, draw the graph of $y = 3x^3 + x^2 - 7x$ for $-2 \leq x \leq 2$. Use the scale: 2 cm for 1 unit on the x -axis and 2 cm for 5 units on the y -axis. (3 marks)



(c) Use the graph to solve the equation:

(i) $3x^3 + x^2 - 7x - 4 = 0$ (2 marks)

$$\left. \begin{array}{l} y = 3x^3 + x^2 - 7x \\ 0 = -4 + 3x^3 + x^2 - 7x \end{array} \right\} -$$

$$y = 4 \checkmark$$

$$x = \underline{\underline{-1.3, -0.6, 1.6 \checkmark}}$$

(ii) $3x^3 + x^2 - 10x = 0$ (3 marks)

$$\left. \begin{array}{l} y = -7x + x^2 + 3x^3 \\ 0 = -10x + x^2 + 3x^3 \end{array} \right\} -$$

$$y = 3x \checkmark$$

x	0	-2
y	0	-6

$$x = \underline{\underline{-2, 0, 1.7 \checkmark}}$$

21.

- (a) Fadhili deposited Ksh. 400 000 in an account that paid compound interest on deposits at a rate of 7% p.a. At the end of 3 years, he withdrew all the money from the account.

- (i) Calculate the amount that Fadhili withdrew. (2 marks)

$$\begin{aligned}\text{Accumulated amount} &= 400,000 \left(1 + \frac{7}{100}\right)^3 \checkmark \\ &= 400,000 \times 1.225043 \\ &= \text{Ksh. } 490,017.20 \quad \therefore \text{Amount he withdrew is } \underline{\underline{\text{Ksh. } 490,017}} \checkmark\end{aligned}$$

- (ii) Fadhili invested the withdrawn amount in shares. The value of the shares depreciated at a rate of 1.5% every 6 months. Determine the value of the shares at the end of 2 years correct to 2 decimal places. (3 marks)

$$\begin{aligned}A &= P \left(1 - \frac{r}{100}\right)^n; n = 2 \times 2 = 4 \checkmark \\ \text{Amount} &= 490,017.20 \left(1 - \frac{1.5}{100}\right)^4 \checkmark \\ &= 490,017.20 \times 0.94133655 \\ &= \underline{\underline{\text{Ksh. } 461,217.10}} \checkmark\end{aligned}$$

- (iii) Determine the gain or loss from Fadhili's investments in the 5 years. (1 mark)

Negative if loss, positive if gain.

$$\begin{aligned}\text{Gain/loss} &= \text{Amount at the end of 5 years} - \text{Initial amount} \\ &= \text{Ksh. } (461,217.10 - 400,000) \\ &= \underline{\underline{\text{Ksh. } 61,217.10}} (\text{gain}) \checkmark\end{aligned}$$

- (b) Nyambuto invested Ksh. 400 000 in a financial institution that paid compound interest at the rate of 6% per annum. After n years, the amount had accumulated to Ksh. 500 000. Calculate the value of n , correct to the nearest whole number. (4 marks)

$\text{From } A = P \left(1 + \frac{r}{100}\right)^n \Rightarrow \left(1 + \frac{r}{100}\right)^n = \frac{A}{P}$ Taking logs on both sides; $n \log \left(1 + \frac{r}{100}\right) = \log A - \log P$ $\therefore n = \frac{\log A - \log P}{\log \left(1 + \frac{r}{100}\right)} \checkmark$	$\begin{aligned}n &= \frac{\log 500000 - \log 400000}{\log \left(1 + \frac{6}{100}\right)} \checkmark \\ &= \frac{5.69897 - 5.60206}{0.0253059} \checkmark \\ &= 3.8295 \\ &\approx \underline{\underline{4 \text{ years}}} \checkmark\end{aligned}$
--	--



22. The probabilities of obtaining scores 1, 2, 3, 4 and 5 using a biased pentagonal spinner were recorded as shown in the following table.

Score	1	2	3	4	5
Probability	k	0.1	0.25	$2k$	0.2

(a) Determine:

- (i) the value of k . (2 marks)

$$k + 0.1 + 0.25 + 2k + 0.2 = 1 \checkmark$$

$$3k + 0.55 = 1$$

$$3k = 1 - 0.55$$

$$3k = 0.45 \Rightarrow k = \frac{0.45}{3} = 0.15 \checkmark$$

- (ii) the probability of obtaining a score of 4. (1 mark)

$$P(\text{score} = 4) = 2k$$

$$= 2 \times 0.15 = 0.3 \checkmark$$

(b) The spinner was spun twice.

- (i) Work out the probability of obtaining an even number in the first spin and an odd number in the second spin. (4 marks)

		First spin				
		1	2	3	4	5
Second spin	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)

✓✓ 2 marks

Probability space = 25
Favourable outcomes = 6 } ✓

$$P(\text{even and odd}) = \frac{6}{25} \text{ or } 0.24 \checkmark$$

Alternatively:

$$P(2,1) \text{ OR } P(2,3) \text{ OR } P(2,5) \text{ OR } P(4,1) \text{ OR } P(4,3) \text{ OR } P(4,5) \checkmark$$

$$P(2,1) + P(2,3) + P(2,5) + P(4,1) + P(4,3) + P(4,5)$$

$$(0.1 \times 0.15) + (0.1 \times 0.25) + (0.1 \times 0.2) + (0.3 \times 0.15) + (0.3 \times 0.25) + (0.3 \times 0.2) \checkmark$$

$$0.015 + 0.025 + 0.02 + 0.045 + 0.075 + 0.06 \checkmark$$

$$0.24 \checkmark$$

- (ii) The total score, S , for the two spins was obtained. Determine the probability that $S \geq 9$.

(3 marks)

$$P(4,5) \text{ OR } P(5,4) \text{ OR } P(5,5) \checkmark$$

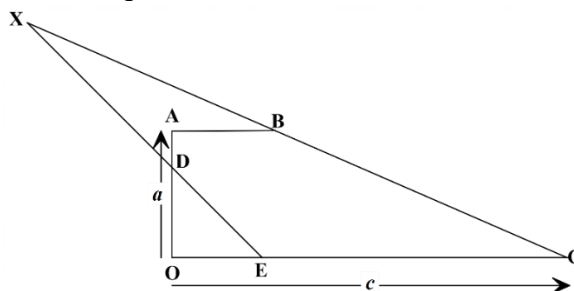
$$P(4,5) + P(5,4) + P(5,5)$$

$$(0.3 \times 0.2) + (0.2 \times 0.3) + (0.2 \times 0.2) \checkmark$$

$$0.06 + 0.06 + 0.04$$

$$\underline{\underline{0.16 \checkmark}}$$

23. In the figure below, OABC is a trapezium. **AB** is parallel to **OC** and **OC = 4 AB**. D is point on **OA** such that **OD : DA = 3 : 1** and E is a point on **OC** such that **OE : EC = 1 : 3**.



- (a) Given that **OC = c** and **OA = a**, express in terms of **a** and **c**:

(i) **ED**.

(1 mark)

$$\begin{aligned}\mathbf{ED} &= \mathbf{OD} - \mathbf{OE} \\ &= \frac{3}{4}\mathbf{OA} - \frac{1}{4}\mathbf{OC} \\ &= \boxed{\frac{3}{4}\mathbf{a} - \frac{1}{4}\mathbf{c}} \text{ or } \boxed{\frac{1}{4}(3\mathbf{a} - \mathbf{c})} \checkmark\end{aligned}$$

(ii) **CB**.

(1 mark)

$$\begin{aligned}\mathbf{CB} &= \mathbf{OB} - \mathbf{OC} \\ &= \mathbf{OA} + \mathbf{AB} - \mathbf{OC} \\ &= \mathbf{OA} + \frac{1}{4}\mathbf{OC} - \mathbf{OC} \\ &= \mathbf{a} + \frac{1}{4}\mathbf{c} - \mathbf{c} \\ &= \boxed{\mathbf{a} - \frac{3}{4}\mathbf{c}} \text{ or } \boxed{\frac{1}{4}(4\mathbf{a} - 3\mathbf{c})} \checkmark\end{aligned}$$

- (b) Line ED and CB produced intersect at X such that **EX = hED** and **CX = kCB**, where **h** and **k** are scalars.

(i) Express **EX** in terms of **a**, **c** and **h**.

(1 mark)

$$\mathbf{EX} = h\left(\frac{3}{4}\mathbf{a} - \frac{1}{4}\mathbf{c}\right) = \boxed{\frac{3}{4}h\mathbf{a} - \frac{1}{4}h\mathbf{c}} \text{ or } \boxed{\frac{1}{4}h(3\mathbf{a} - \mathbf{c})} \checkmark$$

(ii) Express **CX** in terms of **a**, **c** and **k**.

(1 mark)

$$\mathbf{CX} = k\left(\mathbf{a} - \frac{3}{4}\mathbf{c}\right) = \boxed{k\mathbf{a} - \frac{3}{4}k\mathbf{c}} \text{ or } \boxed{\frac{1}{4}k(4\mathbf{a} - 3\mathbf{c})} \checkmark$$

- (c) Determine the values of **h** and **k**.

(5 marks)

Expressing **OX** in 2 ways:

$$\mathbf{OX} = \mathbf{OE} + \mathbf{EX}$$

$$= \frac{1}{4}\mathbf{c} + \frac{3}{4}h\mathbf{a} - \frac{1}{4}h\mathbf{c}$$

$$= \frac{3}{4}h\mathbf{a} + \left(\frac{1}{4} - \frac{1}{4}h\right)\mathbf{c} \dots (i) \checkmark$$

$$\mathbf{OX} = \mathbf{OC} + \mathbf{CX}$$

$$= \mathbf{c} + k\mathbf{a} - \frac{3}{4}k\mathbf{c}$$

$$= k\mathbf{a} + \left(1 - \frac{3}{4}k\right)\mathbf{c} \dots (ii) \checkmark$$

$$\boxed{k = \frac{3}{4}h} \text{ and } \boxed{1 - \frac{3}{4}k = \frac{1}{4} - \frac{1}{4}h}$$

Substituting for **k** in $\uparrow$

$$1 - \frac{3}{4}\left(\frac{3}{4}h\right) = \frac{1}{4} - \frac{1}{4}h \checkmark$$

$$1 - \frac{9}{16}h = \frac{1}{4} - \frac{1}{4}h$$

$$\frac{5}{16}h = \frac{3}{4} \Rightarrow h = \frac{3}{4} \times \frac{16}{5} = \frac{12}{5} \checkmark$$

$$k = \frac{3}{4} \times \frac{12}{5} = \frac{9}{5} \checkmark$$

Alternatively, express **CX** or **EX** in 2 ways...

$$\left\{ \begin{aligned} \mathbf{CX} &= \mathbf{CE} + \mathbf{EX} \dots (i) \\ &= -\frac{1}{4}(3+h)\mathbf{c} + \frac{3}{4}h\mathbf{a} \\ \mathbf{CX} &= k\mathbf{a} - \frac{3}{4}k\mathbf{c} \dots (ii) \end{aligned} \right\}$$

$$\left\{ \begin{aligned} \mathbf{EX} &= \mathbf{EC} + \mathbf{CX} \\ &= k\mathbf{a} + \left(\frac{3}{4} - \frac{3}{4}k\right)\mathbf{c} \dots (i) \\ \mathbf{EX} &= \frac{3}{4}h\mathbf{a} - \frac{1}{4}h\mathbf{c} \dots (ii) \end{aligned} \right\}$$

- (d) Determine the ratio **ED : DX**.

(1 mark)

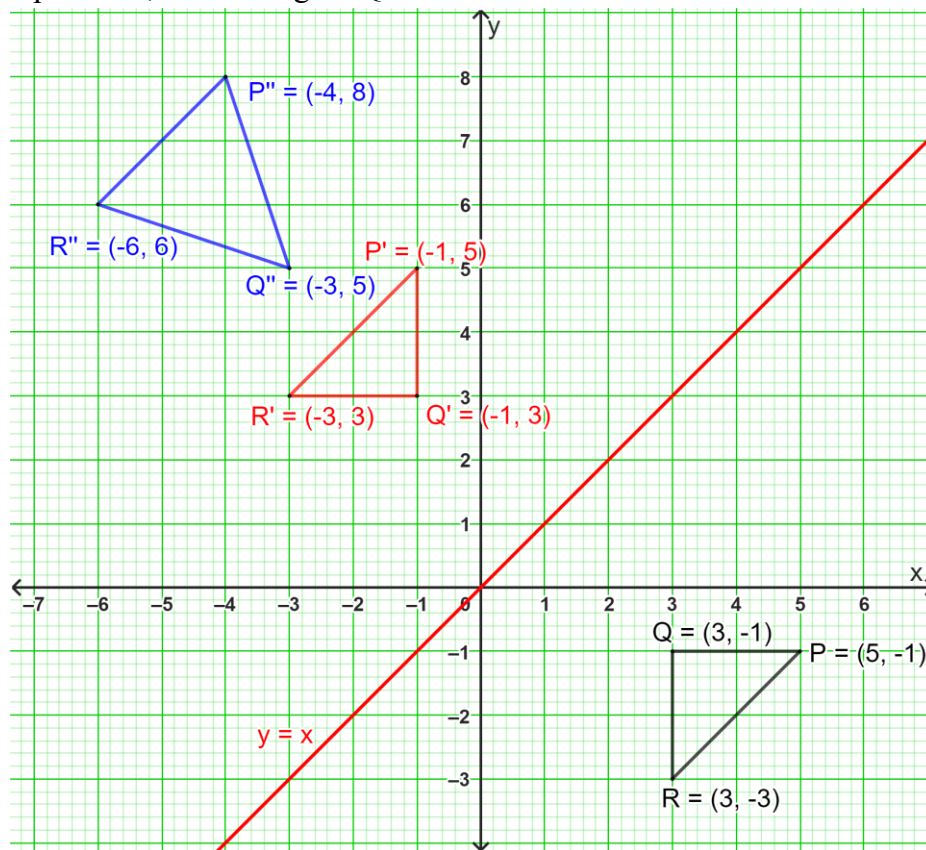
$$\mathbf{DX} = \frac{7}{20}(3\mathbf{a} - \mathbf{c}) \text{ and } \mathbf{ED} = \frac{1}{4}(3\mathbf{a} - \mathbf{c}) \Rightarrow \frac{\mathbf{ED}}{\mathbf{DX}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$$

$$\therefore \mathbf{ED : DX} = \mathbf{5 : 7} \checkmark$$

24. PQR is a triangle with vertices P(5, -1), Q(3, -1) and R(3, -3).

(a) On the grid provided, draw triangle PQR.

(1 mark)



(b) On the same grid, draw $\Delta P'Q'R'$ the image of ΔPQR under a reflection in the line $y = x$.

(3

marks)

$X(a, b) \xrightarrow[y=x]{\text{Reflect}} X'(b, a)$, use the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or construct.

$$\begin{array}{ccc} P & Q & R \\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 3 & 3 \\ -1 & -1 & -3 \end{pmatrix} & = & \begin{pmatrix} (0 \times 5) + (1 \times -1) & (0 \times 3) + (1 \times -1) & (0 \times 3) + (1 \times -3) \\ (1 \times 5) + (0 \times -1) & (1 \times 3) + (0 \times -1) & (1 \times 3) + (0 \times -3) \end{pmatrix} \checkmark = \begin{pmatrix} -1 & -1 & -3 \\ 5 & 3 & 3 \end{pmatrix} \\ \therefore P'(-1, 5), Q'(-1, 3), R'(-3, 3) \checkmark \end{array}$$

(c) $\Delta P''Q''R''$ is the image of $\Delta P'Q'R'$ under a transformation matrix $T = \begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix}$.

(i) Find the coordinates of $P''Q''R''$.

(2 marks)

$$\begin{array}{ccc} P' & Q' & R' \\ \begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix} \begin{pmatrix} -1 & -1 & -3 \\ 5 & 3 & 3 \end{pmatrix} & = & \begin{pmatrix} (1.5 \times -1) + (-0.5 \times 5) & (1.5 \times -1) + (-0.5 \times 3) & (1.5 \times -3) + (-0.5 \times 3) \\ (-0.5 \times -1) + (1.5 \times 5) & (-0.5 \times -1) + (1.5 \times 3) & (-0.5 \times -3) + (1.5 \times 3) \end{pmatrix} \checkmark \\ & = & \begin{pmatrix} -4 & -3 & -6 \\ 8 & 5 & 6 \end{pmatrix} \\ \therefore P''(-4, 8), Q''(-3, 5), R''(-6, 6) \checkmark \end{array}$$

(ii) On the same grid, draw triangle $P''Q''R''$.

(1 mark)

(d) Determine a single transformation matrix that maps ΔPQR directly to $\Delta P''Q''R''$.

(3 marks)

$$\begin{pmatrix} 1.5 & -0.5 \\ -0.5 & 1.5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \checkmark = \begin{pmatrix} (1.5 \times 0) + (-0.5 \times 1) & (1.5 \times 1) + (-0.5 \times 0) \\ (-0.5 \times 0) + (1.5 \times 1) & (-0.5 \times 1) + (1.5 \times 0) \end{pmatrix} \checkmark = \begin{pmatrix} -0.5 & 1.5 \\ 1.5 & -0.5 \end{pmatrix} \checkmark$$