CANDIDATE NAME:	....
INDEX NUMBER:	....
CENTRE CODE:	....
CENTRE NAME:	....
RANDOM NUMBER:	....

121220251011

121/2

KCSE 2025 MATHEMATICS PAPER 2 16 NOV 2025 121/2 MARKING SCHEME

Candidate's signature: .... Date: ....

Random Number: 121220251011

Completed

KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education

121/2

MATHEMATICS Alt. A

Paper 2

Nov. 2025 – 2½ hours

FINAL PROPOSED MARKING SCHEME

Confidential

Candidate's signature: .... **Date:** ....

Instructions to candidates

(a) Confirm that this question paper has your name, name of your school and the correct index number.
(b) Sign and write the date of examination in the spaces provided above.
(c) The paper consists of **two** sections. **Section I** and **Section II.**
(d) Answer **ALL** the questions in Section I and any **FIVE** questions in Section II.
(e) **Show all the steps in your calculations, giving your answer at each stage in the spaces provided below each question.**
(f) Marks may be given for correct working even if the answer is wrong.
(g) **Non-programmable** silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.
(h) ***This paper consists of 19 printed pages.***
(i) ***Candidates must check the question paper to ascertain that all pages are printed as indicated and that no questions are missing***
(j) **Candidates should answer the questions in English.**

For Examiner's Use Only

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**Total**

SECTION II

17	18	19	20	21	22	23	24	**Total**

Grand Total

Final

Turn over

SECTION I (50 marks)

*Answer **all** the questions in this section in the spaces provided.*

1. The first three terms of an arithmetic progression (AP) are $8-x$, $2x$ and $3x+2$. Find the value of x and hence the common difference of the AP. (3 marks)

$d=(3x+2)-2x=x+2\ldots(i)$

$d=2x-(8-x)=3x-8\ldots(ii)$

$Equating: x+2=3x-8$ ✓

$2x=10$

$x=5$ ✓

$d=5+2=7$ **or** $3(5)-8=7$ ✓

OR

$d=(3x+2)-2x=2x-(8-x)$

Solving $(3x+2)-2x=2x-(8-x)$ ✓

$3x+2-2x=2x-8+x$

$2x=10$

$x=5$ ✓

$d=5+2=7$ **or** $3(5)-8=7$ ✓

2. The temperature in a room was measured as 20°C in the morning and 25°C in the afternoon. The difference in the temperature of the room was calculated. Determine the maximum possible error in the calculation. (3 marks)

Least unit of measurement $=\frac{1}{2}\times 1$

$=0.5$

Maximum error $=25.5-19.5=6°\text{C}$ ✓

Minimum error $=24.5-20.5=4°\text{C}$ ✓

Absolute error $=\frac{6-4}{2}=1$ ✓

Alternatively

Actual difference $=25-20=5°\text{C}$

Maximum difference $=25.5-19.5=6°\text{C}$ ✓

Minimum difference $=24.5-20.5=4°\text{C}$ ✓

Absolute error $=\frac{(6-5)+(5-4)}{2}$

$=1$ ✓

3. Find the value of a and b for which $\frac{7\sqrt{2}}{5-3\sqrt{2}}=a+b\sqrt{2}$. (3 marks)

Multipling the numerator & denominator by the conjugate $(5+3\sqrt{2})$ *of the denominator*:

$$\frac{7\sqrt{2}}{5-3\sqrt{2}}\times\frac{5+3\sqrt{2}}{5+3\sqrt{2}}=\frac{35\sqrt{2}+42}{5^2-(3\sqrt{2})^2}\checkmark$$

$$=\frac{42+35\sqrt{2}}{25-18}$$

$$=\frac{42+35\sqrt{2}}{7}$$

$$=6+5\sqrt{2}\checkmark$$

Completed

$\therefore a=6,\ b=5$ ✓ *for both*

4.

(a) Expand and simplify the expression $\left(2+\dfrac{1}{2x}\right)^5$. (2 marks)

$$1\left(2^5\left(\frac{1}{2x}\right)^0\right)+5\left(2^4\left(\frac{1}{2x}\right)^1\right)+10\left(2^3\left(\frac{1}{2x}\right)^2\right)+10\left(2^2\left(\frac{1}{2x}\right)^3\right)+5\left(2^1\left(\frac{1}{2x}\right)^4\right)+1\left(2^0\left(\frac{1}{2x}\right)^5\right)✓$$

$$1(32\times1)+5\left(16\left(\frac{1}{2x}\right)\right)+10\left(8\left(\frac{1}{4x^2}\right)\right)+10\left(4\left(\frac{1}{8x^3}\right)\right)+5\left(2\left(\frac{1}{16x^4}\right)\right)+1\left(1\left(\frac{1}{32x^5}\right)\right)$$

$$\therefore\left(2+\frac{1}{2x}\right)^5=32+\frac{40}{x}+\frac{20}{x^2}+\frac{5}{x^3}+\frac{5}{8x^4}+\frac{1}{32x^5}✓$$

(b) Use the first four terms of the expansion in (a) to estimate the value of $(2.05)^5$. (2 marks)

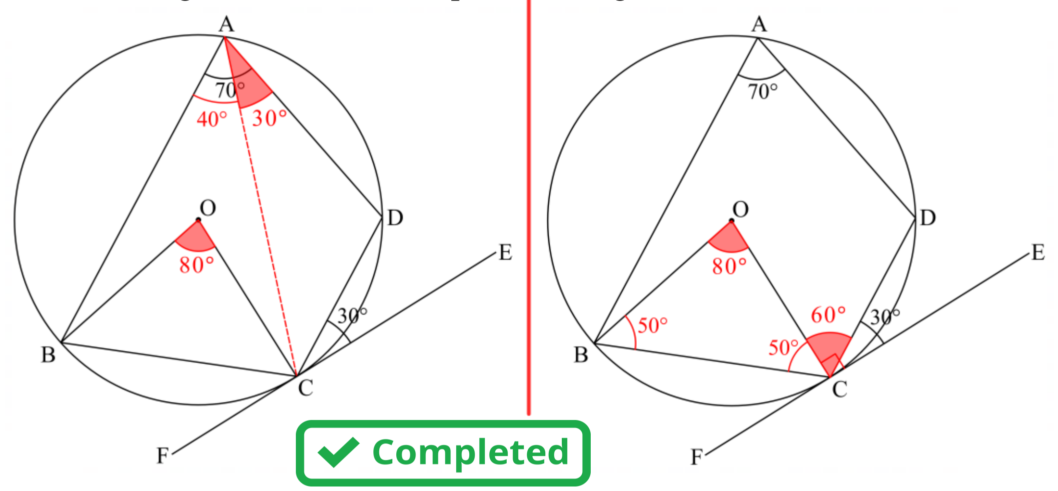
$$2.05=2+0.05 \text{ hence } \frac{1}{2x}=0.05\Rightarrow x=\frac{1}{2\times0.05}=10✓$$

$$(2.05)^5=32+\frac{40}{10}+\frac{20}{10^2}+\frac{5}{10^3}$$

$$=32+4+0.2+0.005$$

$$=36.205✓$$

5. In the following figure, A, B, C and D are points on the circumference of a circle centre, O. Line FE is a tangent to the circle at point C. Angle DCE = 30° and $\angle BAD=70°$.



Giving reasons at each stage, determine the size of the acute angle BOC. (3 marks)

$\angle ECD=\angle DAC=30°$
(alternate segment theorem)✓

$\angle BAC=70°-30°=40°$✓
$\angle BAD=\angle BAC+\angle CAD$

Acute $\angle BAC=2\times40°=80°$✓
($\angle$ subtended by chord BC at O is twice the $\angle$ it subtends at A)

$\angle BCD=180°-70°=110°$
(Opposite $\angle$s of a cyclic quad are supplementary)✓
$\angle OCE=90°$($\angle$ between a tangent and radius)✓
$\therefore\angle OCD=90°-30°=60°$
$\angle OCB=110°-60°=50°$
$\angle OBC=\angle OCB=50°$ (base $\angle$s of an isosceles Δ are equal)
Acute $\angle BAC=180°-(50°+50°)=80°$
(Sum of $\angle$s in a triangle is 180°)✓

6. Two quantities y and x are such that y varies directly as the square of $(x+1)$. Given that $y = 200$ when $x = 4$, determine the equation connecting the two quantities. (3 marks)

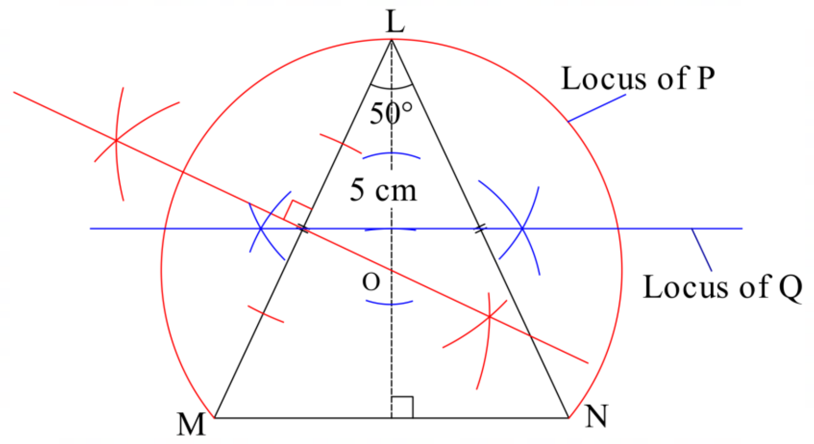
$$y \alpha (x+1)^2 \Rightarrow y = k(x+1)^2 ✓$$

$$k = \frac{y}{(x+1)^2}$$

$$\Rightarrow k = \frac{200}{(4+1)^2} = 8 ✓$$

$$\therefore y = 8(x+1)^2 ✓ \text{ or } y = 8x^2 + 16x + 8$$

7. The following figure shows an isosceles triangle MLN. The height of the triangle is 5 cm and $\angle MLN = 50°$.



On the figure, use a ruler and a pair of compasses only to construct:

(a) The locus of point P such that $\angle MPN = 50°$. (2 marks)

Perpendicular bisector of LM or LN intersects that of the angle bisector of $\angle MLN$ at O.✓
Use centre O and radius OM/ON/OL to draw a major arc MLN.✓

(b) Locate point Q such that the area of Δ MQN is half that of Δ MLN and $\angle MQN = 50°$. (2 marks)

$$\text{Area } MQN = \frac{1}{2} \text{ of } \left(\frac{1}{2} \times 4.7 \times 5\right) = 5.875 \text{ cm}^2$$

$$\perp h \text{ of } \Delta MQN = \frac{5.875}{0.5 \times 4.7} = 2.5 \text{ cm}$$

Construct a parallel to line MN; 2.5 cm away. ✓

Completed

8. A point U is 9 000 nm to the East of Point $T(0°, 50°W)$. Find the longitude at U.

(3 marks)

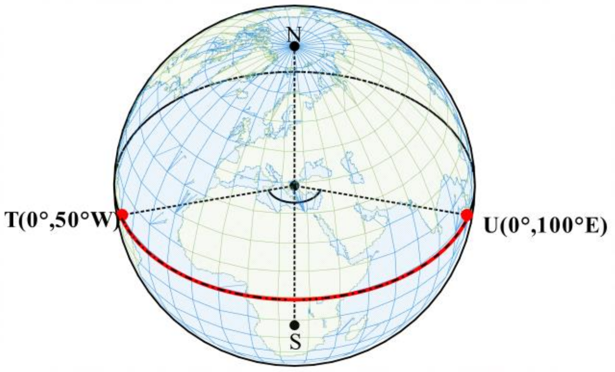
distance $= 60\theta$ *nm*; $\theta = $ *longitude difference*

$\therefore 60\theta = 9{,}000$ ✓

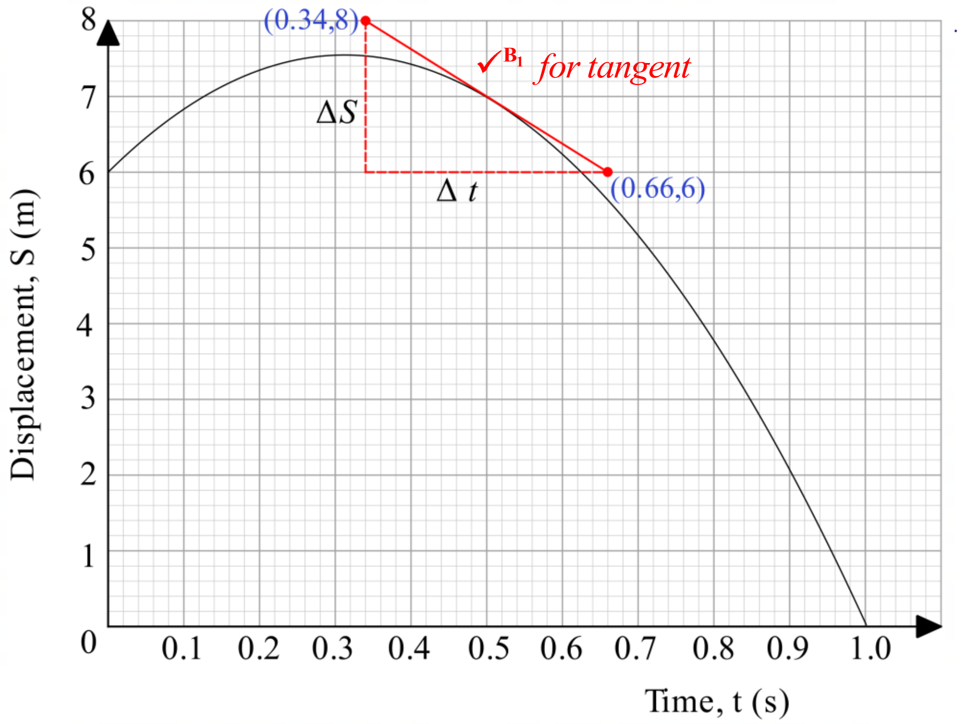
$\theta = \frac{9{,}000}{60} = 150°$ ✓

Longitue of $U = 150° - 50°$

$= 100°E$ ✓



9. The following graph shows the displacement S metres of a moving particle from a point O after time, t seconds $(0 \leq t \leq 1)$.



Use the graph to determine the rate of change of displacement S at time t = 0.5 seconds.

(3 marks)

$$rate = \frac{\Delta S}{\Delta t}$$

$$= \frac{(6-8)m}{(0.66-0.34)s} \checkmark$$

$$= \frac{-2m}{0.32s}$$

$$= -6.25\ m/s \checkmark$$

Completed

10. The ages of 32 residents at a home for the elderly people are presented in the following frequency distribution table.

Age (years)	70 – 74	75 – 79	80 – 84	85 – 89	90 – 94	95 – 99
No. of residents	5	6	7	8	4	2

Calculate the upper quartile (Q_3) age of the residents. (3 marks)

cf	**5**	**11**	**18**	**26**	**30**	**32**

Position of $Q_3 = \frac{3}{4} \times 32 = 24 \;\therefore Q_3$ lies in the class $85 - 89$ with $f = 8$ and $lcb = 84.5$.

i of this class is: $89.5 - 84.5 = 5$.

$$Q_3 = 84.5 + 5\left(\frac{24-18}{8}\right)$$

$$= 88.25$$

11. A circle centre $(-2,3)$ and radius 5 units is drawn on a Cartesian plane. Determine the x intercepts of the circle. (3 marks)

$(x-h)^2 + (y-k)^2 = r^2;\ h = -2,\ k = 3.$

$(x+2)^2 + (y-3)^2 = 5^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 - 25 = 0$

$x^2 + y^2 + 4x - 6y - 12 = 0$; **eqn of the circle**

At $x-$*intercepts,* $y = 0$ *hence*:

$x^2 + 4x - 12 = 0$

$x^2 - 2x + 6x - 12 = 0$

$x(x-2) + 6(x-2) = 0$

$(x+6)(x-2) = 0$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ or $x = 2.$

Alternatively

$(x-h)^2 + (y-k)^2 = r^2;\ h = -2,\ k = 3.$

At $x-$*intercepts,* $y = 0$ *hence*:

$(x+2)^2 + (-3)^2 = 5^2$

$x^2 + 4x + 4 + 9 - 25 = 0$

$x^2 + 4x - 12 = 0$

$x^2 - 2x + 6x - 12 = 0$

$x(x-2) + 6(x-2) = 0$

$(x+6)(x-2) = 0$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ or $x = 2.$

12. An investor deposited Ksh. 20 000 in an account that paid compound interest at a rate of 2.5% every 6 months. At the end of the investment period, the interest earned was Ksh. 5 600. Determine the duration in years of the investment period. (3 marks)

A = Ksh. $(20\ 000 + 5\ 600)$ = Ksh. 25 600

There are 2 interest periods in each year...

$$\therefore 25\ 600 = 20\ 000\left(1 + \frac{2.5}{100}\right)^{2n};\ n = \text{years}$$

$$1.28 = 1.025^{2n}$$

Taking log on both sides:

$$(2n)\log 1.025 = \log 1.28$$

$$n = \frac{\log 1.28}{2\log 1.025} = 4.9987$$

$$\simeq 5 \text{ years}$$

13. Points $A(-4,7)$, $B(4,1)$ and $C(16,-8)$ lie on a straight line. Determine the ratio in which B divides AC. (3 marks)

$$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = \begin{pmatrix}16\\-8\end{pmatrix} - \begin{pmatrix}-4\\7\end{pmatrix} = \begin{pmatrix}20\\-15\end{pmatrix}$$

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}-4\\7\end{pmatrix} = \begin{pmatrix}8\\-6\end{pmatrix} \checkmark$$

Let $\mathbf{AB} = h\mathbf{AC}$.

$$h\begin{pmatrix}20\\-15\end{pmatrix} = \begin{pmatrix}8\\-6\end{pmatrix}$$

$$20h = 8 \Rightarrow h = \frac{2}{5}$$

$$-15h = -6 \Rightarrow h = \frac{2}{5} \checkmark$$

Thus, $\mathbf{AB} = \frac{2}{5}\mathbf{AC} \Rightarrow \mathbf{BC} = \frac{3}{5}\mathbf{AC}$

$$\frac{AB}{BC} = \frac{2/5}{3/5} = \frac{2}{3}$$

$\therefore$ B divides AC (internally) in the ratio; AB:BC $= 2:3$. ✓

$$\frac{m}{m+n}\mathbf{OC} + \frac{n}{m+n}\mathbf{OA} = \mathbf{OB}$$

$$\frac{m}{m+n}\begin{pmatrix}16\\-8\end{pmatrix} + \frac{n}{m+n}\begin{pmatrix}-4\\7\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix} \checkmark$$

$$\frac{16m}{m+n} - \frac{4n}{m+n} = 4 \quad \textbf{or} \quad \frac{-8m}{m+n} + \frac{7n}{m+n} = 1$$

$$16m - 4n = 4m + 4n \quad \textbf{or} \quad -8m + 7n = m + n$$

$$12m = 8n \checkmark \qquad -9m = -6n \checkmark$$

$$\frac{m}{n} = \frac{8}{12} = \frac{2}{3} \qquad \frac{m}{n} = \frac{-6}{-8} = \frac{2}{3}$$

B divides AC in the ratio $m{:}n = 2:3$. ✓

14. A transformation matrix $\mathbf{T} = \begin{pmatrix}a+1 & 4\\4 & a+1\end{pmatrix}$ maps a triangle PQR of area 0.5 unit squares onto a triangle $P'Q'R'$ of area 4.5 unit squares. Determine the possible values of a. (3 marks)

$$asf = |\det| \Rightarrow \det = \pm\frac{4.5}{0.5} = \pm 9$$

$$\therefore \quad (a+1)^2 - (4\times 4) = \pm 9 \checkmark$$

When $\det = 9$:

$$a^2 + 2a + 1 - 16 = 9$$
$$a^2 + 2a - 24 = 0$$
$$a^2 + 6a - 4a - 24 = 0 \checkmark$$
$$a(a+6) - 4(a-6) = 0$$
$$(a-4)(a+6) = 0$$
$$a - 4 = 0,\ a + 6 = 0$$
$$a = 4 \textbf{ or } a = -6 \checkmark$$

When $\det = -9$:

$$a^2 + 2a + 1 - 16 = -9$$
$$a^2 + 2a - 6 = 0 \checkmark$$

Using the quadratic formula:

$$a = \frac{-2 \pm \sqrt{2^2 - (4\times 1\times -6)}}{2\times 1} \checkmark$$

$$a = \frac{-2 \pm \sqrt{28}}{2}$$

$$a = \boxed{\sqrt{7} - 1} \text{ or } \boxed{a = -\sqrt{7} - 1} \checkmark$$

15. A particle starts from a point O and moves in a straight line. Its velocity V m/s at time t seconds is given by $V = 4 - t$. The distance S of the particle from O at time $t = 2$ seconds is 7 m. Calculate S when $t = 4$ seconds. (3 marks)

$$S = \int (4 - t)\, dt = 4t - \tfrac{1}{2}t^2 + c \checkmark$$

When $t = 2$; $\quad 4(2) - \tfrac{1}{2}(2)^2 + c = 7$

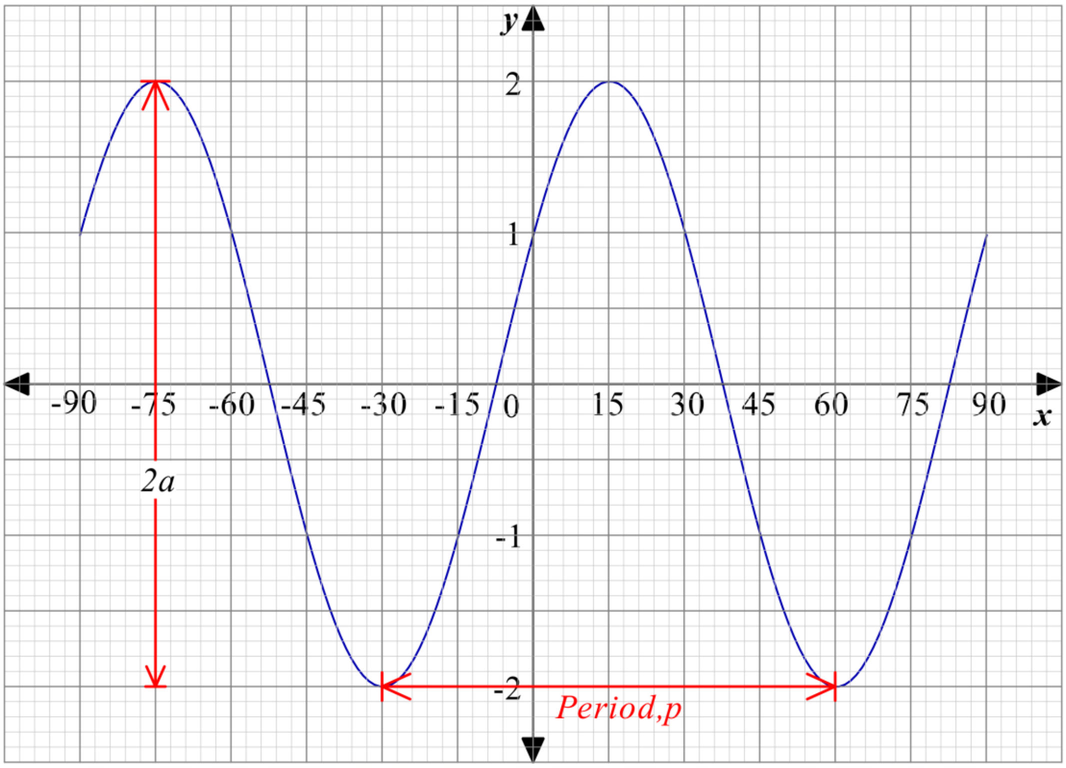
$$c = 7 - 8 + 2$$
$$= 1 \checkmark$$

$$\therefore \boxed{S = 4t - \tfrac{1}{2}t^2 + 1}$$

When $t = 4$; $\quad S = 4(4) - \tfrac{1}{2}(4)^2 + 1 = 9\ m \checkmark$

Completed

16. The following graph represents a wave of the trigonometric function $y = A\sin(\omega x + 30)°$ for $-90° \le x \le 90°$.



Determine the values of scalars A and ω. (3 marks)

$$a = amplitude = \left|\frac{2a}{2}\right| = \frac{2-(-2)}{2} = 2$$

Note: $a \ne -2$. Reason : *The sin wave given rises as it leaves the y - axis just as the fundamental sine wave. It is therefore not a reflection.*

$\therefore A = 2$✓

If it were a reflection, it would not rise as it leaves the y - axis $\Rightarrow$ *A would be* -2.

$$Period, p = 60° - (-30°) = 90°$$ ✓

$$\omega = \frac{360°}{period} = \frac{360°}{90°} = 4$$ ✓

Confidential

SECTION II (50 marks)

*Answer only **five** questions in this section in the spaces provided.*

17. Two varieties of ground nuts type q and type r are sold in the market. Type q is sold at Ksh. 130 per kg while type r is sold at Ksh. 180 per kg.

(a) A trader bought 50 kg of type q and 75 kg of type r from the market. The two varieties were then mixed.

(i) Determine the cost price of 1 kg of the mixture; (2 marks)

$$\text{Cost per kg of mixture} = \frac{(50\times130)+(75\times180)}{50+75} ✓$$

$$= \frac{20000}{125}$$

$$= \text{Ksh. } 160 ✓$$

(ii) The trader sold 80% of the mixture at Ksh. 170 per kg and the rest at Ksh. 180 per kg. Determine the percentage profit made by the trader. (4 marks)

$$\text{Amount from sales} = \left(\frac{80}{100}\times125\times170\right)+\left(\frac{20}{100}\times125\times180\right) ✓$$

$$= \text{Ksh. } 21\,500$$

$$\%\text{ profit} = \frac{21\,500-20\,000}{20\,000}\times100 ✓$$

$$= 7.5\% ✓$$

(b) Another trader bought and mixed the two varieties of ground nuts. The trader made a profit of 25% by selling the mixture at Ksh. 200 per kg. Determine the ratio in which the trader mixed the two varieties. (4 marks)

Option 1

$$\text{SP per kg without profit} = \frac{100}{125}\times200 = \text{Ksh. } 160 ✓$$

Let the ratio $(type\ q):(type\ r) = h:k.$

$$\therefore \frac{130h+180k}{h+k} = 160 ✓$$

$$160h+160k = 130h+180k$$

$$160h-130h = 180k-160k$$

$$30h = 20k ✓$$

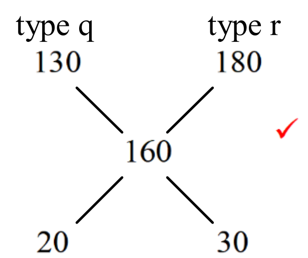
Divide both sides by 30k and simplify:

$$\frac{h}{k} = \frac{2}{3} \Rightarrow h:k = 2:3 ✓$$

Option 2

$$\text{SP per kg without profit} = \frac{100}{125}\times200$$

$$= \text{Ksh. } 160 ✓$$



$$type\ q : type\ r = 20:30 ✓$$

$$= 2:3 ✓$$

18. Registering for a Music club costs Ksh. 7 000 while for a Mathematics club costs Ksh. 7 200. The registration cost is shared equally among members of each club. Originally, both clubs had x members each. However, before payment for registration, 5 Music club members switched to Mathematics club.

(a) Write an expression in terms of x for the amount contributed by:

(i) each Music club member after the switch; (1 mark)

$$\frac{7\ 000}{x-5} \checkmark$$

(ii) each Mathematics club member after the switch. (1 mark)

$$\frac{7\ 200}{x+5} \checkmark$$

(b) Given that a Music club member contributed Ksh. 40 more than a Mathematics club member:

(i) form an equation in x and hence determine the value of x; (5 marks)

$$\frac{7000}{x-5} - \frac{7200}{x+5} = 40 \checkmark$$

Multiply every term by $(x-5)(x+5)$.

$$7000(x+5) - 7200(x-5) = 40(x^2-25)$$

$$7000x + 35000 - 7200x + 36000 = 40x^2 - 1000$$

$$40x^2 + 200x - 72000 = 0$$

$$x^2 + 5x - 1800 = 0 \checkmark$$

$$x^2 + 45x - 40x - 1800 = 0$$

$$x(x+45) - 40(x+45) = 0 \checkmark$$

$$(x+45)(x-40) = 0$$

$x + 45 = 0$ **or** $x - 40 = 0$

$x = -45$ **or** $x = 40 \checkmark$

Ignore -45 hence $x = 40$ members. ✓

(ii) determine the percentage increase in the amount contributed by each remaining Music club member due to the switch. (3 marks)

$$\text{Original amount} = \frac{7\ 000}{40} = \text{Ksh. } 175 \checkmark$$

$$\text{New amount} = \frac{7\ 000}{40-5} = \text{Ksh. } 200 \checkmark$$

$$\%\ \text{increase} = \frac{200-175}{175} \times 100$$

$$= 14\frac{2}{7}\% \checkmark$$

Approved

19. The income tax rates of a certain year were as shown in the following table.

Monthly taxable income in Kenya shillings (Ksh.)	**Tax Rates (%)**
0 – 24 000	10
24 001 – 32 333	25
32 334 – 500 000	30
500 001 – 800 000	32.5
Over 800 000	35

In March that year, Judy's monthly earnings were as follows:

Basic salary	Ksh. 104 644
House allowance	Ksh. 25 000
Commuter allowance	Ksh. 12 000

Judy was entitled to a monthly tax relief of Ksh. 2 400.

(a) Calculate:

(i) Judy's taxable income that month; (2 marks)

$$\text{Income tax} = \text{Basicsaly} + \text{allowances}$$
$$= \text{Ksh. } (104\ 644 + 25\ 000 + 12\ 000) ✓$$
$$= \text{Ksh. } 141\ 644 ✓$$

(ii) The tax payable by Judy that month. (5 marks)

$$\left.\begin{aligned} &\text{Slab 1 tax: } \frac{10}{100} \times 24\ 000 = \text{Ksh. } 2\ 400 \\ &\text{Slab 2 tax: } \frac{25}{100} \times 8\ 333 = \text{Ksh. } 2\ 083.25 \\ &\text{Slab 3 tax: } \frac{30}{100} \times 109\ 311 = \text{Ksh. } 32\ 93.30 \end{aligned}\right\} + ✓ ✓$$

$$\text{Gross tax} = \text{Ksh. } 37\ 276.55 ✓$$

$$\text{Tax payable} = \text{Ksh. } (37\ 276.55 - 2\ 400) ✓$$
$$= \text{Ksh. } 34\ 876.55 ✓$$

(b) In July that year, Judy's basic salary changed, her allowances and monthly tax relief remaining as before. Her net tax that month was Ksh. 39 483.35.

Calculate her new basic salary. (3 marks)

$$\text{Increase in gross tax} = \text{Ksh. } (39\ 483.35 - 34\ 876.55) = \text{Ksh. } 4\ 606.80 ✓$$

July slab 3 (income tax increase) $= x$

$$\frac{30}{100} \times x = \text{Ksh. } 4\ 606.80 \Rightarrow x = \frac{4\ 606.80}{0.3} = \text{Ksh. } 15\ 356 ✓$$

$$\text{July basic salary} = \text{Ksh. } (141\ 644 + 15\ 356 - 37\ 000)$$
$$= \text{Ksh. } 120\ 000 ✓$$

✔ Approved

20.

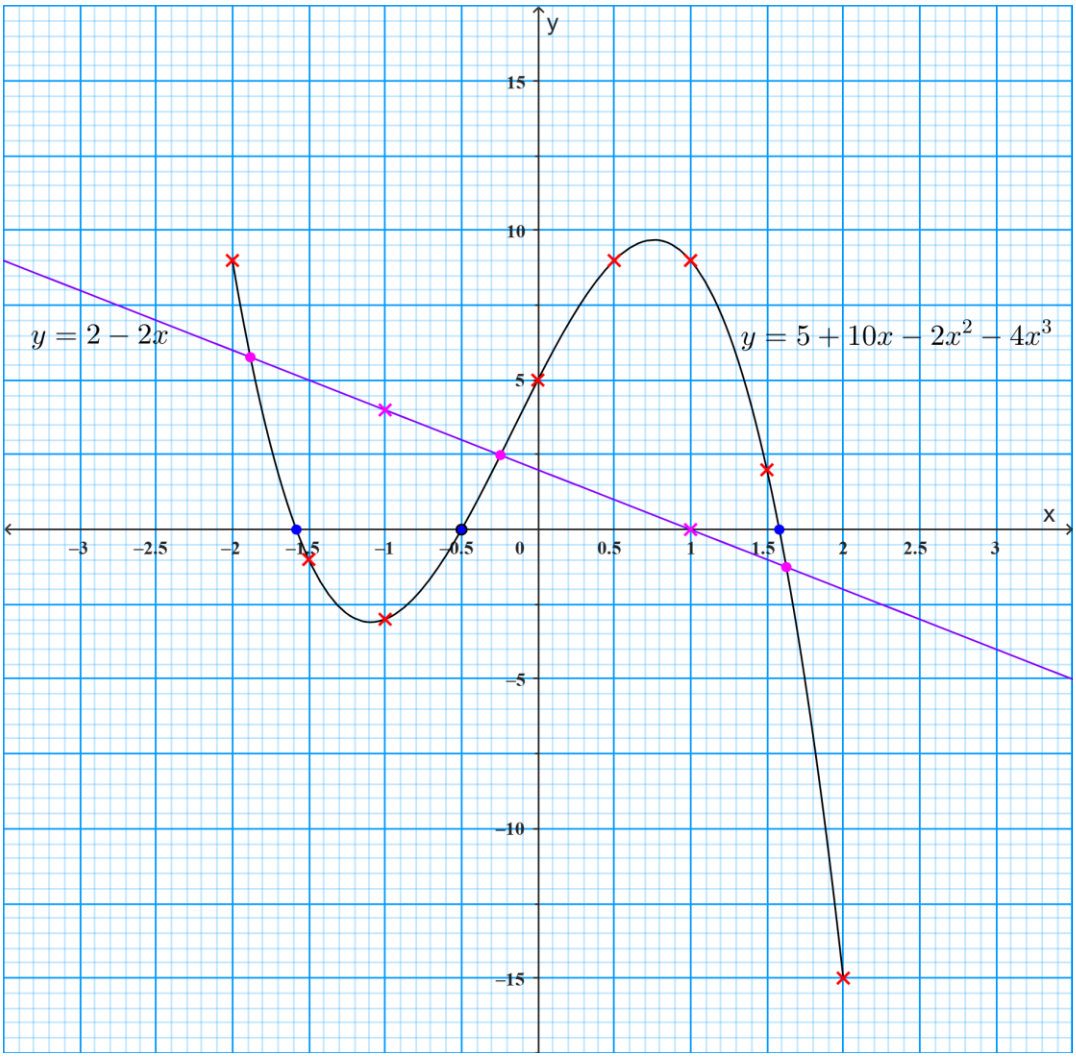
(a) The following table shows values of x and some values of y for the curve $y = 5 + 10x - 2x^2 - 4x^3$ for $-2 \leq x \leq 2$.

Complete the table by filling in the missing values of y.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	**9**	-1	-3	**0**	5	**9**	9	2	**-15**

2 values – 1 mark (2 marks)

(b) On the grid provided, draw the graph of $y = 5 + 10x - 2x^2 - 4x^3$. Use the scale 2 cm to represent 1 unit on x-axis, 2 cm represents 5 units on y-axis. (3 marks)



(c)

(i) Use the graph to solve the equation $5+10x-2x^2-4x^3=0$. (1 mark)

$$\left.\begin{array}{l} y=5+10x-2x^2-4x^3 \\ 0=5+10x-2x^2-4x^3 \end{array}\right\}-$$

$$y=0$$

$$x=-1.6,\ -0.5,\ 1.6\checkmark$$

(ii) By drawing a suitable straight line on the graph, solve the equation:
$4x^3+2x^2-12x-3=0$. (4 marks)

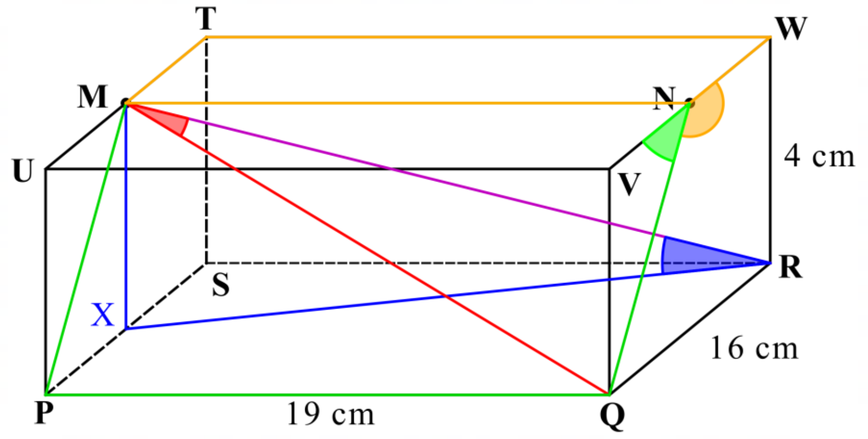
$$\left.\begin{array}{l} y=5+10x-2x^2-4x^3 \\ 0=3+12x-2x^2-4x^3 \end{array}\right\}-$$

$$y=2-2x$$

x	1	−1
y	0	4

$x=-1.9,\ -0.25,\ 1.6\checkmark$

21. The following figure represents a cuboid **PQRSTUVW**. Line PQ = 19 cm, QR = 16 cm and RW = 4 cm. Points **M** and **N** are midpoints of lines UT and VW respectively.



(a) Calculate the length of line RM. (2 marks)

$$RM = \sqrt{RS^2 + SX^2 + M^2}$$
$$= \sqrt{19^2 + 8^2 + 4^2} ✓$$
$$= 21\ cm ✓$$

(b) Calculate correct to 2 decimal places:

(i) the angle between line RM and the plane PQRS; (2 marks)

$$\sin\theta = \frac{XM}{RM}$$
$$\Rightarrow \theta = \sin^{-1}\frac{XM}{RM}$$
$$= \sin^{-1}\left(\frac{4}{21}\right) ✓$$
$$= 10.98° ✓$$

(ii) the angle between lines RM and MQ; (3 marks)

Note: RM = MQ = 21 cm. Take triangle MQR.

Using cosine rule:

$$16^2 = 21^2 + 21^2 - (2 \times 21 \times 21\cos\theta) ✓$$
$$882\cos\theta = 626$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{626}{882}\right) ✓$$
$$= 44.79° ✓$$

OR $2\sin^{-1}\left(\frac{8}{21}\right) = 44.79°$ ✓

(iii) the obtuse angle between planes PMNQ and MNWT. (3 marks)

Acute ∠ QNV:

$$\tan\theta = \frac{VQ}{VN} \Rightarrow \theta = \tan^{-1}\frac{VQ}{VN}$$
$$= \tan^{-1}\left(\frac{4}{8}\right) = 26.57° ✓$$

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$$\text{Obtuse } \angle QNW = 180° - 26.57° ✓$$
$$= 153.43° ✓$$

22. A bag contains five balls randomly numbered 1 to 5. The balls are identical except for colour.

(a) Two balls are randomly drawn from the bag, one at a time without replacement.

(i) Draw a probability space to show all the possible pairs of numbers on the two balls drawn from the bag. (2 marks)

1st draw, x / 2nd draw, y	1	2	3	4	5
1		2,1	3,1	4,1	5,1
2	1,2		3,2	4,2	5,2
3	1,3	2,3		4,3	5,3
4	1,4	2,4	3,4		5,4
5	1,5	2,5	3,5	4,5	

✓2

(ii) Find the probability that both the numbers on the balls drawn from the bag were greater than 3. (1 mark)

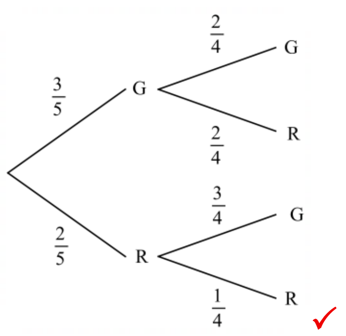
$$P(x > 3, y > 3) = \frac{2}{20} \text{ or } \frac{1}{10} ✓$$

(iii) Find the probability that the sum of the two numbers on the balls drawn does not exceed 6. (2 marks)

$$P(x + y \leq 6) = \frac{12}{20} \text{ or } \frac{3}{5} ✓$$

(b) Three balls in the bag are green in colour and the rest are red. Determine the probability that the two balls drawn in (a) were:

(i) of the same colour; (3 marks)



P(same colour) = P(GG) OR P(RR)

$$= \left(\frac{3}{5} \times \frac{2}{4}\right) + \left(\frac{2}{5} \times \frac{1}{4}\right) ✓$$

$$= \frac{8}{20} ✓ \text{ or } \frac{2}{5}$$

(ii) of mixed colours. (2 marks)

$$P(\text{mixed colours}) = 1 - P(\text{same colour})$$

$$= 1 - \frac{8}{20} ✓$$

$$= \frac{12}{20} \text{ or } \frac{3}{5} ✓$$

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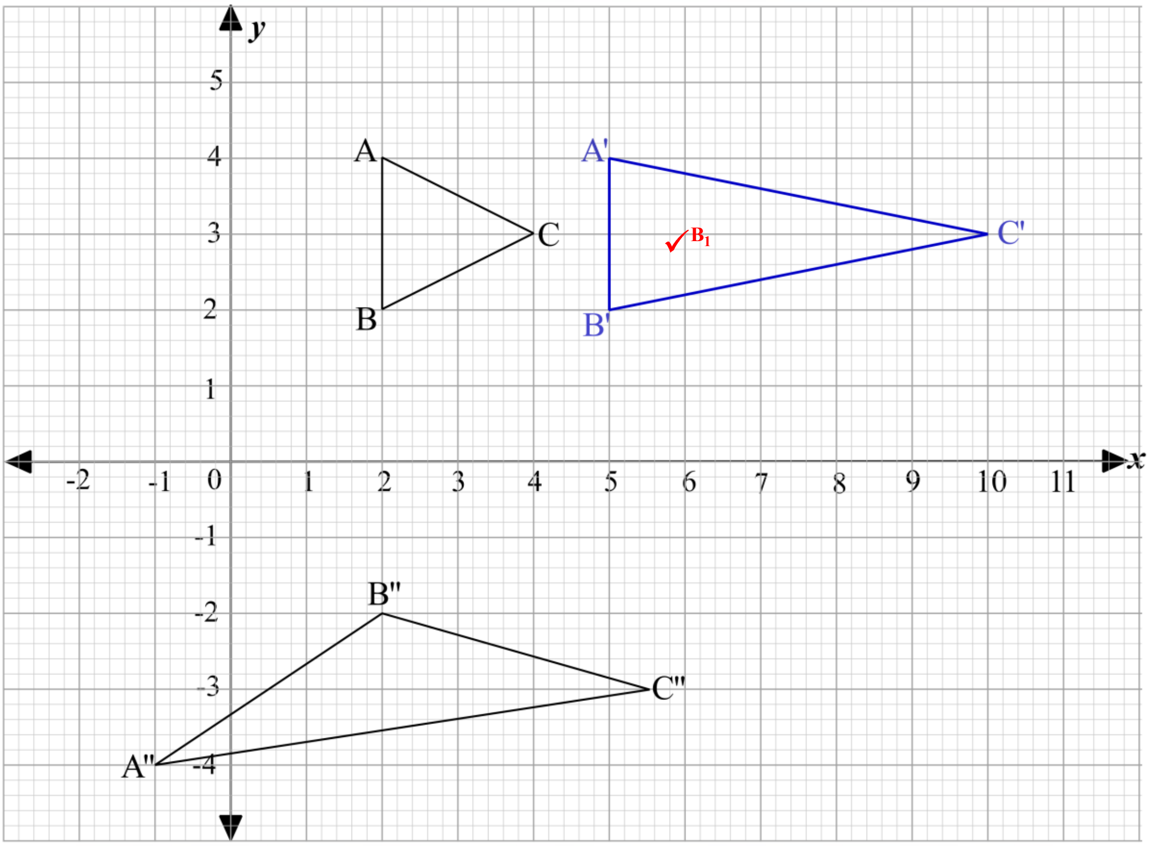
23. Triangle $A''B''C''$ is the image of triangle ABC under transformations $\mathbf{T}_1 = \begin{pmatrix} 2.5 & 0 \\ 0 & 1 \end{pmatrix}$ followed by $\mathbf{T}_2 = \begin{pmatrix} 1 & -1.5 \\ 0 & -1 \end{pmatrix}$. Triangles ABC and $A''B''C''$ are drawn on the following grid.



(a)

(i) On the same grid provided, draw $\Delta\, A'B'C'$, the image of ABC under transformation matrix $\mathbf{T}_1 = \begin{pmatrix} 2.5 & 0 \\ 0 & 1 \end{pmatrix}$. (3 marks)

$$\begin{pmatrix} 2.5 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 2 & 4 \\ 4 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 2.5\times2+0\times4 & 2.5\times2+0\times2 & 2.5\times4+0\times3 \\ 0\times2+1\times4 & 0\times2+1\times2 & 0\times4+1\times3 \end{pmatrix} ✓$$

$$= \begin{pmatrix} 5 & 5 & 10 \\ 4 & 2 & 3 \end{pmatrix}$$

$A'(5,4),\ B'(5,2),\ C'(10,3)$ ✓

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(ii) Describe fully the transformation represented by matrix $\mathbf{T_1}$. (2 marks)

A stretch with *y*-axis invariant (parallel to the *x*-axis)✓ and scale factor 2.5.✓

(b)

(i) Find the single transformation matrix that maps Δ ABC onto Δ A″B″C″. (2 marks)

$$\begin{pmatrix} 1 & -1.5 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 2.5 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1\times 2.5 + {}^-1.5\times 0 & 1\times 0 + {}^-1.5\times 1 \\ 0\times 2.5 + {}^-1\times 0 & 0\times 0 + {}^-1\times 1 \end{pmatrix} ✓$$

$$= \begin{pmatrix} 2.5 & -1.5 \\ 0 & -1 \end{pmatrix} ✓$$

(ii) Determine the area of Δ A″B″C″. (3 marks)

Area of $\Delta ABC = \frac{1}{2}\times 2\times 2 = 2$ sq. units✓

$$\det\begin{pmatrix} 2.5 & -1.5 \\ 0 & -1 \end{pmatrix} = (2.5\times -1) - (0\times -1.5) = -2.5$$

$asf = |-2.5| = 2.5$✓

Area of $\Delta A''B''C'' = 2\times 2.5 = 5$ sq. units✓

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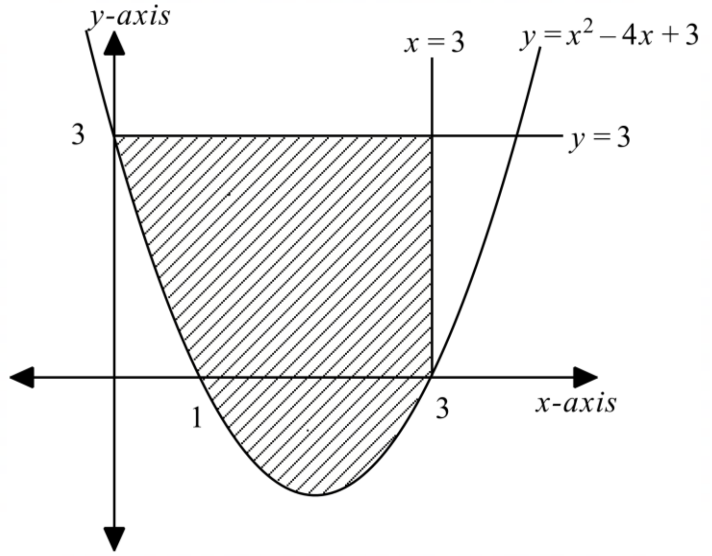
24. The following figure is a sketch of the curve whose equation is $y = x^2 - 4x + 3$. The $y-$intercept of the curve is at $y = 3$ while the $x-$intercepts are at $x = 1$ and $x = 3$. The region bounded by the curve, the line $y = 3$ and the line $x = 3$ is shaded.



(a)

(i) Evaluate $\int_0^1 \left(x^2 - 4x + 3\right)\,dx$ (3 marks)

$$\int_0^1 \left(x^2 - 4x + 3\right)\,dx = \left[\frac{x^3}{3} - 2x^2 + 3x + c\right]_0^1 \checkmark$$

$$= \left(\frac{1^3}{3} - 2(1)^2 + 3(1) + c\right) - \left(\frac{0^3}{3} - 2(0)^2 + 3(0) + c\right) \checkmark$$

$$= \frac{4}{3} + c - 0 - c$$

$$= 1\frac{1}{3} \checkmark$$

(ii) Calculate the area of the shaded region above the x-axis. (2 marks)

$$\text{Area} = (3 \times 3) - 1\frac{1}{3} \checkmark$$

$$= 7\frac{2}{3} \text{ sq. units} \checkmark$$

Alternatively

$$Area = \int_0^3 (3)\,dx - 1\frac{1}{3} \checkmark$$

$$= [3x + c]_0^3 - 1\frac{1}{3}$$

$$= (3(3) + c) - (3(0) + c) - 1\frac{1}{3} \checkmark$$

$$= 9 + c - 0 - c - 1\frac{1}{3}$$

$$= 7\frac{2}{3} \text{ sq. units} \checkmark$$

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(b) Calculate the area of the shaded region below the x-axis. (3 marks)

Since the region is below the x-axis, on integration, a negative value will be obtained. Area cancan only be positive hence the negative sign before the $\int$ sign.

$$Area = -\int_{1}^{3}\left(x^2 - 4x + 3\right)\,dx = -\left[\frac{x^3}{3} - 2x^2 + 3x + c\right]_1^3 ✓$$

$$= -\left(\frac{3^3}{3} - 2(3)^2 + 3(3) + c\right) + \left(\frac{1^3}{3} - 2(1)^2 + 3(1) + c\right) ✓$$

$$= 0 + c + \frac{4}{3} - c$$

$$= 1\frac{1}{3} \text{ sq. units} ✓$$

(c) Calculate the area of the entire shaded region. (2 marks)

$$\text{Total area} = 1\frac{1}{3} + 7\frac{2}{3} ✓$$

$$= 9 \text{ sq. units} ✓$$

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